Reliability Mechanics and Modeling for IC Packaging
– Theory, Implementation and Practices

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Outline

• Introduction
• Temperature Loading
• Mechanical Loading
• Moisture Loading
• Electrical Current Loading - Multi-Physics Modeling
• Summary
• Introduction

• Temperature Loading

• Mechanical Loading

• Moisture Loading

• Electrical Current Loading - Multi-Physics Modeling

• Summary
• Design of a package must consider the interactions among wafer, package, and board (e.g. CPI – chip-package-interaction).
Examples of Failures

- Interface delamination
- Thin film delamination
- Substrate cracking
- Pad cratering
- Solder brittle failure
- Fatigue crack
- Electromigration
- Film rupture
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Thermal-mechanical Stress

Stress-free

Cooling-down

Heating-up

Thermal stress in solder

Cooling-down

Package warpage
Mechanical Load

- Handheld electronic products are susceptible to drop impact failure.
Moisture-induced Failure

Stage 1: Moisture absorption

Device exposed to reflow temperature, typically 260°C

Stage 2: Initiation of delamination

Stage 3: Delamination propagation

Stage 4: Package cracking and vapor release
Electrical Current Stressing

$e^-$

Voids

Hillocks

Electron wind
Organization of Course

• **Four modules**
  – Temperature loading
    ▪ Thermal mismatch, temperature gradient, etc.
  – Mechanical loading
    ▪ Drop and impact, bend, etc.
  – Moisture loading
    ▪ Moisture diffusion, vapor pressure, swelling.
  – Electrical current loading - combined loading
    ▪ Electrical-thermal, electrical-thermal-mechanical, and electromigration (electrical-thermal-mechanical-mass transport)

• **Theory, implementation, and best practices.**
• Introduction
• Temperature Loading
• Mechanical Loading
• Moisture Loading
• Electrical Current Loading - Multi-Physics Modeling
• Summary
Outline

• Basic concepts and analytical solutions

• Applications
  – Die-level thermal stress – thermal stress in TSV
  – Package-level thermal stress problem – warpage
  – Chip-package interaction (CPI) – sub-modeling technique
  – Board level thermal stress problems
    ▪ Solder ball thermal cycling
    ▪ Creep equations

• Best practices
  – Initial stress free condition
  – Full model vs. global/local model
  – Volume averaging
  – Stress singularity
Stress, Strain, and Basic Elastic Material Properties

\[ \sigma = \frac{P}{A_0} \]
\[ \delta = L - L_0 \]
\[ \varepsilon = \frac{\delta}{L_0} \]
\[ \nu = -\frac{\varepsilon_{lat}}{\varepsilon_{long}} \]

Hooke’s law

\[ \sigma = E\varepsilon \]

Material | Elastic Modulus (GPa) | Poisson Ratio |
--- | --- | ---
Adhesive | 0.01 – 10 | 0.4 – 0.49 |
Epoxy | 10 – 30 | 0.35 – 0.48 |
Copper | 120 | 0.35 |
Silicon | 131 | 0.28 |
Solder | 25 – 51 | 0.35 |
Substrate/PCB | 11 – 26 | various |
Coefficient of Thermal Expansion (CTE)

- Material dimensions change with temperature.
- The rate of dimensional change with temperature is called CTE.
- Primary driver of thermo-mechanical stress in electronic packages.

\[
\varepsilon^{th} = \frac{\Delta L}{L}
\]

\[
\alpha = \frac{\epsilon^{th}}{\Delta T}
\]

<table>
<thead>
<tr>
<th>Material</th>
<th>( \alpha ) (ppm/°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum</td>
<td>23</td>
</tr>
<tr>
<td>Stainless Steel</td>
<td>12</td>
</tr>
<tr>
<td>Silicon</td>
<td>2.6</td>
</tr>
<tr>
<td>Copper</td>
<td>17</td>
</tr>
<tr>
<td>Underfill Epoxy</td>
<td>20, 70</td>
</tr>
<tr>
<td>FR4 (in plane)</td>
<td>15, 10</td>
</tr>
<tr>
<td>FR4 (z-axis)</td>
<td>65, 180</td>
</tr>
</tbody>
</table>

Increase internal energy (heat) causes atomic bond lengths to increase.
Thermal Mismatch – Two-Layer Structure

Superposition: axial loading + bending

$T = T_{\text{dep}}$

$T = T_{\text{room}}$

$\alpha_2 < \alpha_1$

unconstrained

constrained

released
Thermal-Mismatch – Multi-Layer Structure

Superposition: axial loading + bending
**Thermal-Mismatch : Analytical Solution**

Multi-layer structure

\[
\varepsilon_{xi}(y) = (y - h_b) K + C, \text{ with } h_{i-1} \leq y \leq h_i.
\]

\[
\sigma_{xi}(y) = Y_i [(y - h_b) K + C + \nu_i A - \eta_i \varepsilon_i^0], \text{ with } h_{i-1} \leq y \leq h_i.
\]

- \( h_b \) is the Bending Axis of the composite device;
- \( C \) is the Uniform Strain of the composite device;
- \( A \) is the Axial constant Strain of the composite device;
- \( K \) is the Curvature of the composite device;
- \( K_{nat} \) is the component due to Natural bending of the curvature of the composite device;
- \( K_{app} \) is the Applied curvature of the composite device;
- \( \varepsilon_i^0 \) is the Stress-free Strain of layer \( i \);

\[
K = K_{nat} + K_{app} = \sum_{i=1}^{n} Y_i D_i [h_{mi} - h_b] (\eta_i \varepsilon_i^0 - C - \nu_i A) + K_{app}
\]

\[
\varepsilon_i^0 = \alpha_i (T_{room} - T_{depi}) + \varepsilon_i^{Btin}
\]

Thermal Mismatch - Shear Stress and Strain on Solder Joint

A simplified and approximate solution

Cooling to a lower temperature

(PCB shrinks more than package)

$$\gamma = \frac{L(\alpha_{PCB} - \alpha_{chip}) \Delta T}{h}$$

$$\tau = G \cdot \gamma$$

\(\tau\): shear stress in solder
\(\gamma\): solder ball shear strain
\(G\): elastic shear modulus
\(L\): distance of netural point (half-die size, diagonal)
\(h\): solder ball stand-off
\(\alpha\): CTE
Thermal Mismatch – TSV Structure

Shrink fit of compound cylinder

\[ \delta = \frac{bp}{E_o} \left( \frac{b^2 + c^2}{c^2 - b^2} + \nu_o \right) + \frac{bp}{E_i} \left( \frac{a^2 + b^2}{b^2 - a^2} - \nu_i \right) \]

\[ \sigma_r = \frac{a^2 p_i - b^2 p_o}{b^2 - a^2} - \frac{(p_i - p_o)a^2 b^2}{(b^2 - a^2)r^2} \]

\[ \sigma_0 = \frac{a^2 p_i - b^2 p_o}{b^2 - a^2} + \frac{(p_i - p_o)a^2 b^2}{(b^2 - a^2)r^2} \]

\[ u = \frac{1 - \nu}{E} \left( \frac{a^2 p_i - b^2 p_o}{b^2 - a^2} \right) + \frac{1 + \nu}{E} \left( \frac{p_i - p_o}{b^2 - a^2} \right) \]

Single cylinder under external and internal pressure

Thermal Stress due to Temperature Gradient

\[
\sigma_r = \alpha E \left[ -\frac{1}{r^2} \int_a^b T r \, dr + \frac{r^2 - a^2}{r^2(b^2 - a^2)} \int_a^b T r \, dr \right]
\]

\[
\sigma_\theta = \alpha E \left[ -T + \frac{1}{r^2} \int_a^b T r \, dr + \frac{r^2 + a^2}{r^2(b^2 - a^2)} \int_a^b T r \, dr \right]
\]

\[
\sigma_r = \alpha E \left[ \frac{1}{b^2} \int_0^b T r \, dr - \frac{1}{r^2} \int_0^r T r \, dr \right]
\]

\[
\sigma_\theta = \alpha E \left[ -T + \frac{1}{b^2} \int_0^b T r \, dr + \frac{1}{r^2} \int_0^r T r \, dr \right]
\]
Finite Element Analysis (FEA)

• Analytical solutions are usually not applicable to complex problems.
• Based on dividing the problem into discrete elements for which analytical solutions exit.
• Analytical equations are then formed into a matrix and solved simultaneously.
• Method may require multiple iterations to converge and may have to divide “time” steps to reach a solution.
• Used for structural, thermal, EMI, fluid dynamics, diffusion, stock analysis, etc.

Bracket meshed into 11,295 elements.
Application – Warpage Analysis

(a) Die Attaching
(b) Underfill Dispensing and Curing
(c) Warpage Control by Stiffener
(d) Warpage Control by Lid

Bare Die FCBGA
FCBGA with Stiffener Ring
FCBGA with Lid
Capped-Die Flip Chip Package Design - assembly process
Using an underfill-like encapsulant, a metallic cap is attached to the die.
Comparison for Warpage Control Methods by FEA

a) Bare die on substrate
b) Package with stiffener
c) Package with lid
d) Capped-die package
Experimental Verification - Warpage

Chip-Package Interaction (CPI)
Multilevel Sub-modeling Technique

(a) package level

(b) critical solder level

(c) die-solder interface level

(d) interconnect level

Xuefeng Zhang, Se Hyuk Im, Rui Huang, and Paul S. Ho, Chip-Package Interaction and Reliability Impact on Cu/Low-k Interconnects, Chapter 2, UT Austin 2008/
Multilevel Sub-modeling Technique

- Multilevel models are chained to obtain the driving force for delamination.
- Thousands of lines in ANSYS APDL codes have been written for the model.
- Typical model has one million DOF and takes a few hours to solve.

XH Liu, TM Shaw, G Bonilla - Advanced Metallization Conference, 2010 - sematech.org
Temperature Cycling
Temperature Cycling

- Temperature cycling – uniform temperature condition assumed
**Power Cycling**

- Temperature gradient exists.
- Much less severe than temperature cycling.
Fatigue Life Estimate

\[ W_p = \int_0^{\varepsilon_p} \sigma d\varepsilon_p \]

\[ N_f = A (\Delta W_p)^{-n} \]

Coffin-Manson Equation
Solder Creep Equations (ANAND Model)

- Anand’s model (rate-dependent visco-plastic model)

\[
\frac{d\varepsilon_p}{dt} = A \left[ \sinh \left( \frac{\xi \sigma}{s} \right) \right]^{-\frac{1}{m}} \exp \left( -\frac{Q}{RT} \right)
\]

\[
\frac{ds}{dt} = \left\{ h_0 \left( \frac{|B|}{B} \right)^\alpha \frac{B}{|B|} \right\} \frac{d\varepsilon_p}{dt}
\]

\[
B = 1 - \frac{s}{s^*}
\]

\[
s^* = \hat{s} \left[ \frac{d\varepsilon_p}{dt} \frac{A}{\exp \left( -\frac{Q}{RT} \right)} \right]^n
\]

<table>
<thead>
<tr>
<th>Variable/Parameter</th>
<th>Meaning</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s_0 )</td>
<td>initial value of deformation resistance</td>
<td>Stress</td>
</tr>
<tr>
<td>( Q/R )</td>
<td>( Q ) = activation energy ( R ) = universal gas constant</td>
<td>Energy/volume/Energy/(volume temp)</td>
</tr>
<tr>
<td>( A )</td>
<td>pre-exponential factor</td>
<td>1/time</td>
</tr>
<tr>
<td>( \xi )</td>
<td>multiplier of stress</td>
<td>Dimensionless</td>
</tr>
<tr>
<td>( m )</td>
<td>strain rate sensitivity of stress</td>
<td>Dimensionless</td>
</tr>
<tr>
<td>( h_0 )</td>
<td>hardening / softening constant</td>
<td>Stress</td>
</tr>
<tr>
<td>( \hat{s} )</td>
<td>coefficient for deformation resistance</td>
<td>Stress</td>
</tr>
<tr>
<td>( n )</td>
<td>strain rate sensitivity of saturation</td>
<td>Dimensionless</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>strain rate sensitivity of hardening or softening</td>
<td>Dimensionless</td>
</tr>
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</table>
# Anand Constants for Various Solder Alloys

<table>
<thead>
<tr>
<th>Parameter</th>
<th>SAC105</th>
<th>SAC305</th>
<th>SAC387</th>
<th>SAC405</th>
<th>SnPb</th>
<th>SnPbAg</th>
<th>SnAg</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_0$</td>
<td>2.3479</td>
<td>18.07</td>
<td>3.2992</td>
<td>20</td>
<td>12</td>
<td>12.41</td>
<td>39.09</td>
</tr>
<tr>
<td>$Q/R$</td>
<td>8076</td>
<td>9096</td>
<td>9883</td>
<td>10561</td>
<td>9200</td>
<td>9400</td>
<td>8900</td>
</tr>
<tr>
<td>$A$</td>
<td>3.773</td>
<td>3484</td>
<td>15.773</td>
<td>325</td>
<td>4.2E6</td>
<td>4E6</td>
<td>2.23E4</td>
</tr>
<tr>
<td>$\xi$</td>
<td>0.9951</td>
<td>4</td>
<td>1.0673</td>
<td>10</td>
<td>1.5</td>
<td>1.5</td>
<td>6</td>
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<tr>
<td>$m$</td>
<td>0.4454</td>
<td>0.2</td>
<td>0.3686</td>
<td>0.32</td>
<td>0.3</td>
<td>0.303</td>
<td>0.182</td>
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<tr>
<td>$h_0$</td>
<td>4507.5</td>
<td>144000</td>
<td>1076.9</td>
<td>8E5</td>
<td>1.4E3</td>
<td>1379</td>
<td>3321.15</td>
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<tr>
<td>$\hat{s}$</td>
<td>3.5833</td>
<td>26.4</td>
<td>3.1505</td>
<td>42.1</td>
<td>14</td>
<td>13.79</td>
<td>73.81</td>
</tr>
<tr>
<td>$n$</td>
<td>0.012</td>
<td>0.01</td>
<td>0.0352</td>
<td>0.02</td>
<td>0.071</td>
<td>0.07</td>
<td>0.018</td>
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<tr>
<td>$\alpha$</td>
<td>2.1669</td>
<td>1.9</td>
<td>1.6832</td>
<td>2.57</td>
<td>1.31</td>
<td>1.3</td>
<td>1.82</td>
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</tbody>
</table>

SAC105 D. Bhafe et al., *Constitutive Behavior of Sn3.8Ag0.7Cu and Sn1.0Ag0.5Cu Alloys at Creep and Low Strain Rate Regimes*, IEEE Transactions on Components and Packaging Technologies, Vol. 31, No. 3, September 2008, p. 622.

SAC305 Motalab et al., *Determination of Anand Constants for SAC Solders using Stress-Strain or Creep Data*, ITherm 2012, pp. 909-921.

SAC387 D. Bhafe et al., *Constitutive Behavior of Sn3.8Ag0.7Cu and Sn1.0Ag0.5Cu Alloys at Creep and Low Strain Rate Regimes*, IEEE Transactions on Components and Packaging Technologies, Vol. 31, No. 3, September 2008, p. 622.

SAC405 W. Qiang et al., *Experimental determination and modification of Anand model constants for Pb-free material 95.5Sn4.0Ag0.5Cu*, in EUROSIIME Conf. IEEE, 2007.


# Solder Creep Equations

<table>
<thead>
<tr>
<th>Solder alloys</th>
<th>$C_1$ (1/sec)</th>
<th>$C_2$ (1/Pa)</th>
<th>$C_3$</th>
<th>$C_4$ (°K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>95.5Sn-3.9Ag-0.6Cu</td>
<td>441000</td>
<td>5x10⁻⁹</td>
<td>4.2</td>
<td>5412</td>
</tr>
<tr>
<td>63Sn-37Pb</td>
<td>$926(508 - T)/T$</td>
<td>$1/(37.78x106 - 74414T)$</td>
<td>3.3</td>
<td>6360</td>
</tr>
</tbody>
</table>

$$\frac{\partial \varepsilon}{\partial t} = C_1 \left[ \sinh \left( C_2 \sigma \right) \right]^{C_3} \exp \left( -\frac{C_4}{T} \right)$$

$C_1$, $C_2$, $C_3$, $C_4$ are the input constants for ANSYS finite element program. The stress unit is Pa.

Solder Creep Equations (NEW)

<table>
<thead>
<tr>
<th>Solder alloys</th>
<th>$C_1$ (1/sec)</th>
<th>$C_2$ (1/Pa)</th>
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<th>$C_4$ (°K)</th>
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<tbody>
<tr>
<td>95.5Sn-3.9Ag-0.6Cu</td>
<td>441000</td>
<td>5x10^{-9}</td>
<td>4.2</td>
<td>5412</td>
</tr>
<tr>
<td>Sn(3.5-3.9)Ag(.5-.8)Cu</td>
<td>500000</td>
<td>10x10^{-9}</td>
<td>5.0</td>
<td>5807</td>
</tr>
<tr>
<td>Fraunhofer’s LF</td>
<td>277984</td>
<td>24.47x10^{-9}</td>
<td>6.41</td>
<td>6504</td>
</tr>
</tbody>
</table>

\[
\frac{\partial \varepsilon}{\partial t} = C_1 \left[ \sinh \left( C_2 \sigma \right) \right]^{C_3} \exp \left( -\frac{C_4}{T} \right)
\]

$C_1$, $C_2$, $C_3$, $C_4$ are the input constants for ANSYS finite element program. The stress unit is Pa.

Lau, J., and W. Dauksher, “Creep Constitutive Equations of Sn(3.5-3.9)wt%Ag(0.5-0.8)wt%Cu Lead-Free Solder Alloys”, in *Micromaterials and Nanomaterials*, edited by B. Michel, IZM, Berlin, Germany, 2004, pp. 54-62.
Solder Creep Equations

SnPb Creep Equation

\[ \dot{\varepsilon} = \frac{\sigma}{E} + B_1 D \left( \frac{\sigma}{E} \right)^3 + B_2 D \left( \frac{\sigma}{E} \right)^7 \]

where
- \( \dot{\varepsilon} \) = Total Strain Rate (1/sec)
- \( \sigma \) = Stress (MPa)
- \( E \) = Modulus of Elasticity (MPa) = \((56000 - 88T)\)
- \( T \) = Temperature (K)
- \( B_1 = 1.70 \times 10^{12} \) 1/sec
- \( B_2 = 8.90 \times 10^{24} \) 1/sec
- \( D = \exp \left( \frac{-5413}{T} \right) \)

SnAgCu Creep Equation

\[ \dot{\varepsilon} = \frac{\sigma}{E} + A_1 D_1 \left( \frac{\sigma}{\sigma_n} \right)^3 + A_2 D_2 \left( \frac{\sigma}{\sigma_n} \right)^{12} \]

where
- \( \dot{\varepsilon} \) = Total Strain Rate (1/sec)
- \( \sigma \) = Stress (MPa)
- \( E \) = Modulus of Elasticity (MPa) = \((59533 - 66.667T)\)
- \( T \) = Temperature (K)
- \( A_1 = 4.0 \times 10^{-7} \) 1/sec
- \( A_2 = 1.0 \times 10^{-12} \) 1/sec
- \( D_1 = \exp \left( -3223/T \right) \)
- \( D_2 = \exp \left( -7348/T \right) \)
- \( \sigma_n = 1 \) MPa
Best Practice (1) - Stress-Free Setting

• Three most commonly used initial stress-free temperatures
  – The solidus temperature of solder material (e.g., for SnAgCu, this temperature is 217°C)
  – The room temperature as initial stress-free (e.g. 25°C)
  – The high dwell temperature of thermal cycle or operating conditions (denoted as $T_{\text{max}}$, e.g. =125°C for thermal cycling from -25°C to 125°C)

• $T_{\text{max}}$ as stress-free condition is recommended.

Best Practice (2) – One Global Model (Full Model)

- Coarse element at most of the places
- Refined element at critical solder joints
- No tie element is used
- Refined solder balls consider both SMD and MD pads
Best Practice (2) – Sub-modeling (Global/Local)

Substrate/underfill interface
Comparison - Global/Local Model and Full Model

- The global/local modeling approach provides satisfactory results for the volumetrically averaged and maximum values.
- For some location, the difference in maximum value can be as high as 20% (not shown here).

- “Full Model” is recommended.
$\Delta W_{ave} = \sum \Delta W_i \frac{V_i}{V_i}$

- $\Delta W_{ave}$ = average inelastic strain energy density accumulated per cycle for fixed thickness layer elements
- $\Delta W_i$ = strain energy density accumulated per cycle for each element $i$
- $V_i$ = volume of each element $i$

- **Volume averaging over a fixed thickness of thin layer is recommended.**
Best Practice (4) - Worst Case Solder Joint Location

– For FC-BGA package
  ▪ It’s commonly accepted that the worst case solder joint is located in the outermost along diagonal direction under the die shadow.

– Metrics
  ▪ Von Mises stress, creep strain, creep strain energy density, peel stress
  ▪ Maximum value, averaged value

– Results
  ▪ Averaged per-cycle creep strain and stress shows the worst-case solder joint at the middle of die edge
  ▪ The maximum value of these two parameters show the worst joint at the die corner
  ▪ Peel stress finds the maximum tensile stress at the joint one row inside from the die shadow corner

• Experimental data
  – The solder joint one row inside from the corner of die shadow usually has the highest crack growth rate
  – All solder joints along the die edge have comparable crack growth rates

• Correlation with experimental data is recommended.

General Results: SnPb vs. SnAgCu

Maximum Von Mises Stress (MPa)

Temperature range -25 to +100°C

Maximum Equiv Creep Strain
Wafer Level Package (Fan-in WLP) – Worst Location

\[ \Delta W_{ave} = \frac{\sum \Delta w \cdot v}{\sum v} \]
WLP - von Mises Stress Map in Solder Balls

1/8th model

Package center

Package corner

Critical solder ball
WLP - Plastic Work Density Map

1/8th model

Stress Singularities in Material Joints

Summary

• Thermal mismatch vs. temperature gradient

• Analytical solution
  – Layered structure (stress, warpage, effective CTE)
  – Cylindrical structure (TSV)

• Die-level thermal stress – thermal stress in TSV

• Package-level thermal stress problem – warpage

• Chip-package interaction (CPI) – submodeling technique

• Board level thermal stress problem
  – Solder ball thermal cycling (Flip chip BGA, WLP)
  – Creep equations
  – Best method for practice
    ▪ Initial stress free condition; full model vs. global/local model; worst solder ball location, volume averaging

• Stress singularity of joint materials
• Introduction

• Temperature Loading

• Mechanical Loading

• Moisture Loading

• Electrical Current Loading - Multi-Physics Modeling

• Summary
Outline

• JEDEC drop test standard (old and new)

• Simulation method
  – Input G
  – Large mass
  – Input D
  – Direct acceleration
  – Global/local method

• 4-point bending test
JEDEC Drop Test Set-Up

- Peak acceleration: 1500g
- Impulse time: 0.5ms with half-sine shape
## JEDEC Test Standard JESD22-B111 (Old)

<table>
<thead>
<tr>
<th>Group</th>
<th>Number Components</th>
<th>Component Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>4</td>
<td>U1, U5, U11, &amp; U15</td>
</tr>
<tr>
<td>B</td>
<td>4</td>
<td>U2, U4, U12, &amp; U14</td>
</tr>
<tr>
<td>C</td>
<td>2</td>
<td>U6 &amp; U10</td>
</tr>
<tr>
<td>D</td>
<td>2</td>
<td>U7 &amp; U9</td>
</tr>
<tr>
<td>E</td>
<td>2</td>
<td>U3 &amp; U13</td>
</tr>
<tr>
<td>F</td>
<td>1</td>
<td>U8</td>
</tr>
</tbody>
</table>
JEDEC Test Standard JESD22-B111A (New, 11/2016)
\{M\}[\ddot{u}_1] = \{C\}[\dot{u}_1] + \{K\}[u_1] = 0
with initial conditions
\[ [u_1]|_{t=0} = 0, \quad [\dot{u}_1]|_{t=0} = \sqrt{2gH} \]
and boundary condition
\[ a = \begin{cases} 
1500g \sin\frac{\pi t}{t_w} & \text{for } t \leq t_w, \quad t_w = 0.5 \\
0 & \text{for } t \geq t_w 
\end{cases} \]
Large Mass Method

Large mass $10^8$ times mass of structure coupled (1mm away from PCB bottom)

Acceleration = $1500 \times g \times \sin (\pi \times t/0.5)$
Force = $10^8 \times$ Acceleration
FEA Modeling: Input-D Method

\{M\}[\ddot{u}_1] = \{C\}[\dot{u}_1] + \{K\}[u_1] = 0

with initial conditions

\[ [u_1]_{t=0} = 0, \quad [\ddot{u}_1]_{t=0} = \sqrt{2gH} \]

and boundary condition

\[ [u_1]_{\text{at hole}} = \begin{cases} 
- \left( \frac{t_w}{\pi} \right)^2 (1500g) \sin \left( \frac{\pi t}{t_w} \right) + \left( \frac{t_w}{\pi} (1500g) + \sqrt{2gH} \right) t, & t \leq t_w \\
\left( 2 \frac{t_w}{\pi} (1500g) + \sqrt{2gH} \right) t - \left( \frac{t_w}{\pi} \right)^2 (1500g), & t \geq t_w
\end{cases} \]
FEA Modeling: Direct Acceleration Method

\[
\{M\}[\ddot{u}_2] + \{C\}[\dot{u}_2] + \{K\}[u_2] = \begin{cases} 
-\{M\}1500g \sin \frac{\pi t}{t_w} & t \leq t_w, \ t_w = 0.5 \\
0 & t \geq t_w 
\end{cases}
\]

with initial conditions
\[
[u_1]|_{t=0} = 0, \quad [\dot{u}_1]|_{t=0} = \sqrt{2gH}
\]

and boundary conditions
\[
[u_2]|_{\text{at hole}} = 0
\]

• Since acceleration impulse is given on the screw locations, the displacements are not fixed and the board is moving in one direction.
• Since the displacement is fixed at the screw locations, the board is vibrating as expected.
Distribution of Board Strain in x direction along line AB at t = 1.5 ms

Comparison of acceleration time history

- Direct Acceleration method is recommended, but all methods provide identical results.
FEA Global Model

PCB

Solder Bump Model

Solder Bump Arrays

Solder Layer Model

Solder Layer
FEA Local Model

- Global Model
- Cut Boundary Distance at 2mm x 2mm
- Cut Boundary DOF Constraint
- Cut Boundary DOF Constraints at 2mm x 2mm
- Local Model 1
- Local Model 2
FEA Local Model – Solder Joint

Schematic View of Cu Post WLP

<table>
<thead>
<tr>
<th>Part No.</th>
<th>Part Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>PCB</td>
</tr>
<tr>
<td>2</td>
<td>PCB Pad</td>
</tr>
<tr>
<td>3</td>
<td>Solder Ball</td>
</tr>
<tr>
<td>4</td>
<td>Copper Post</td>
</tr>
<tr>
<td>5</td>
<td>Epoxy</td>
</tr>
<tr>
<td>6</td>
<td>Passivation</td>
</tr>
<tr>
<td>7</td>
<td>Silicon Die</td>
</tr>
</tbody>
</table>

FE Model of Detailed Solder Ball

Critical Failure Region

Meshing Critical Failure Region (10 μm Layer)
Effect of Cut Boundaries in Sub-modeling

![Graph showing the effect of cut boundaries on maximum peeling stress. The x-axis represents cut boundary distances in millimeters (1mm x 1mm, 1.5mm x 1.5mm, 2mm x 2mm, 2.5mm x 2mm), and the y-axis represents maximum peeling stress in MPa. The graph shows that as the cut boundary distance increases, the maximum peeling stress remains relatively constant.]
Global Model Results – Frequency and Mode Shape

Mode 1: 212 Hz
Mode 2: 551 Hz
Mode 3: 915 Hz
Mode 4: 1257 Hz
Mode 5: 2193 Hz
Experimental Validation

Strain (Micro-strain, $\varepsilon_x$)

Time (ms)

Strain (Micro-strain, $\varepsilon_y$)

Time (ms)
Board Strain Analysis: Package Corner Strain ($\varepsilon_x$)

- $U1 > U3 > U8 > U2 > U7 > U6$.
- $U8$ bends in opposite direction to $U1$. 

Strain History in x Direction

Strain History in y Direction
Ball Stress Contour Plot on ¼\textsuperscript{th} JEDEC Board

- U6: 299 MPa
- U7: 346 MPa
- U8: 486 MPa
- U1: 513 MPa
- U2: 367 MPa
- U3: 464 MPa
**Maximum Peel Stress**

Overall, the stress developed in components can be ranked as:

U1 > U8 > U3 > U2 > U7 > U6

**Failure rank:**
- Group A (U1, U5, U11 and U15) > Group F (U8) > Group E (U3, U13).
Correlation of Solder Joint Stress and Board Strain

Board Strain Ranking: U1 > U3 > U8 > U2 > U7 > U6
Solder Joint Stress Ranking: U1 > U8 > U3 > U2 > U7 > U6

• Exact correlation does not exist for strain and stress values.

4-Point Bending Test

Segment CC'

\[ \varepsilon_x = \frac{3h\delta}{a(3L-4a)} \]

Segment CD

\[ \varepsilon_x = \frac{12h\delta}{L^3} x \]
4-Point Bending Test

- 4PT bending test to take advantage of uniform strain BC for each component.
- JEDEC board + 9 components with electric monitoring for each one of them.
- Cyclic testing of freq 3Hz @ 1.5mm, 2mm and 2.5mm deflections to DC failure.
Cyclic Bending: Modeling Methodology

- Global/local modeling methodology
  - Elastic global model.
  - Local models use EPP SAC405 solder material properties.

M Xie, Solder Reliability Models in 4 Point Bend, Intel, 2006.
Strain Distribution

![Graph showing strain distribution](image-url)

- **Package inside**
- **Package edge**

- **Microstrain, LE11**
  - Linear moment
  - Constant moment

- **Distance from the center**
  - **d=1.5mm**
  - **d=2.0mm**
  - **d=2.5mm**
Board Side or Package Side?

- Consistency with failure propagation in terms of peeling stress
  - Package side: inside-out
  - Board side: outside-in

- Failure happens more likely at the package side: higher failure parameter value.
Failure Pattern

Solder joint crack propagation direction:
• Package side: inside-out
• Board side: outside-in

Sequential Failure (1.5mm deflection)

Simultaneous Failure (2.5mm deflection)

Zhou T and Fan XJ. Effect of system design and test conditions on wafer level package drop test reliability. SMTA International. October 2013.
Summary

• **JEDEC drop test standard**
  – JESD22-B111, old one, with 15 components
  – JESD22-B111A, new one, with 4 components or 1 component

• **Finite element modeling**
  – Input G method, large mass method, input displacement method, direct acceleration method
  – Global/local modeling
  – Peel stress used as indicator for failure

• **Four-point bending test and modeling**
  – Global/local modeling
  – Global with linear elastic but nonlinear geometry analysis
  – Local model with elastic-plastic modeling
• Introduction

• Temperature Loading

• Mechanical Loading

• Moisture Loading

• Electrical Current Loading - Multi-Physics Modeling

• Summary
Outline

- Introduction
- Moisture related reliability testing
- Moisture diffusion modeling
- Vapor pressure theory
- Hygroscopic swelling
Moisture Absorption of Electronic Packages

• Electronic packages absorb moisture in uncontrolled humid conditions prior to the surface mount on board.

Polymer Materials in Electronic Packaging

- **Bulk-form**
  - encapsulation (e.g. mold compound)
  - substrate …

- **Adhesives**
  - die-attach, underfill
  - thermal adhesives …

- **Thick- or thin- film**
  - solder mask
  - passivation

- Polymeric materials are susceptible to moisture absorption.
Moisture Absorption in Polymeric Materials

- **Under 85°C/85%RH condition**, \( \rho_w = \text{Csat} = 2.47 \times 10^{-2} \text{ g/cm}^3 = 81.2 \rho_a \) (Csat: saturated moisture concentration)
  - Moisture is condensed into liquid state.
  - Moisture exists in micro-pores or free volumes (in bulk or at interface).
  - Moisture vaporizes at reflow, possibly still at mixed liquid/vapor phases.

\( \rho_w \): moisture density in polymeric material
\( \rho_a \): ambient moisture density under 85°C/85%RH

Moister condensation in a typical underfill

Moisture Related Testing

• Moisture sensitivity test

Stage 1: Moisture absorption  Stage 2: Soldering reflow

• Highly accelerated stress test (HAST)

• Biased HAST

Basic Concepts of Moisture Diffusion

• What is the Relative Humidity (RH)?
  – Defined as vapor pressure ratio associated with temperature $T$

$$RH = \frac{Actual \; vapor \; pressure \; of \; the \; air}{Saturated \; vapor \; pressure \; of \; the \; air} \times 100\%$$

• Moisture concentration, $C(x, t; T, RH)$
  – Mass of moisture per unit volume of substance.

• Diffusion Coefficient / Diffusivity, $D(T)$
  – Measures the rate of mass diffusion
  – Defined as the amount of mass flux per unit concentration gradient ($m^2/s$)
  – A function of material and temperature

• Saturated Moisture Concentration, $C_{sat}(RH, T)$
  – The maximum mass of moisture per unit volume of the substance $kg/m^3$.

• Solubility, $S(T)$ – Henry’s law
  – The ability of the substance to absorb moisture
  – Defined as the maximum mass of moisture per unit volume of the substance per unit pressure ($kg/(m^3Pa)$).
  – A function of material and temperature

$$S = \frac{C_{sat}}{P}$$

where $P = ambient$ pressure in given RH
Moisture Diffusion Modeling

- **Fickian diffusion theory**

  \[ \frac{\partial C}{\partial t} = -\nabla \cdot (-D_0 \nabla C) \]

  \( C \): Concentration, kg/m\(^3\); \( D_0 \): diffusivity, m\(^2\)/s

- **Discontinuity at interface**

  Concentration discontinuity
  After normalization
## An Overview of Moisture Diffusion Modeling

<table>
<thead>
<tr>
<th>Normalized field variable</th>
<th>( C_{\text{sat}} ): temperature-dependent</th>
<th>RH: time-dependent</th>
<th>Non-Henry’s law</th>
<th>ANSYS</th>
<th>ABAQUS</th>
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<tr>
<td>Galloway et al. (1997)</td>
<td>C/S</td>
<td>✓</td>
<td>x</td>
<td>x</td>
<td>✓</td>
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<tr>
<td>Wong et al. (1998)</td>
<td>C/C(_{\text{sat}})</td>
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<td>x</td>
<td>x</td>
<td>x</td>
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<tr>
<td>Jang et al. (1993)</td>
<td>C/M</td>
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<td>x</td>
<td>x</td>
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<tr>
<td>Wong et al. (2016)</td>
<td>C/C(_{\text{sat}})</td>
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<td>✓</td>
<td>x</td>
<td>x</td>
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<tr>
<td>Markus et al. (2016)</td>
<td>C</td>
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<td>✓</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Chen et al. (2017)</td>
<td>( a_w )</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Ma et al. (2019)</td>
<td>C/K</td>
<td>✓</td>
<td>✓</td>
<td>x</td>
<td>✓</td>
</tr>
</tbody>
</table>

Water Activity Theory

• Water activity $a_w$ is a continuous field variable derived from chemical potential.

• Moisture flux in terms of water activity gradient.
  \[ \mathbf{J}_m = -K_i D_{0,i} \nabla a_w \]

• Moisture concentration is given as
  \[ C = K a_w \]
  $K$: generalized solubility, kg/m$^3$

• Moisture diffusion equation
  \[ \frac{\partial (K_i a_w)}{\partial t} = \nabla \cdot (K_i D_{0,i} \nabla a_w) \]

• Water activity is continuous and no normalization is required for multi-material system.

Current Moisture Diffusion Theory in ANSYS

- \( \bar{C} = \frac{C}{C_{\text{sat}}} \) is used as field variable.

\[
\frac{\partial (C_{\text{sat}} \bar{C})}{\partial t} = \nabla \cdot (D \nabla (C_{\text{sat}} \bar{C})) + G
\]

\[
C_{\text{sat}} \frac{\partial \bar{C}}{\partial t} + \bar{C} \frac{\partial C_{\text{sat}}}{\partial T} \frac{\partial T}{\partial t} = \nabla \cdot \left( [D] C_{\text{sat}} \nabla \bar{C} + \bar{C} \frac{\partial C_{\text{sat}}}{\partial T} \nabla T \right) + G
\]

where
- \( D \) : diffusivity matrix
- \( C (x,y,z,t) \) : concentration
- \( G \): diffusing substance generation rate per unit volume
- \( \nabla \): gradient operator = \( \left\{ \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\} \), \( \nabla \cdot \): divergence operator

- In the above formulation, \( C_{\text{sat}} \) is considered as temperature-dependent only.
- However, \( C_{\text{sat}} \) is both time- and temperature-dependent in general.
Henry’s Law in Moisture Diffusion

- \( C_{sat} = p_{amb} S \)

\( p_{amb} \): ambient partial vapor pressure  
\( p_{amb} = RH \ p_g(T) = RH \ p_0 e^{(-E_p/RT)} \) (\( p_g \) is saturated vapor pressure)

\( S \) : solubility  
\( S(T) = S_0 e^{(E_s/RT)} \)

\[
C_{sat} = RH \ p_0 S_0 e^{(E_s-E_p)/RT} = RH \ K_0 e^{(-E_k/RT)} = RH(t)K(T)
\]

\[
K = K(T) = K_0 \exp \left( -\frac{E_k}{RT} \right)
\]

\[
K_0 = p_0 S_0, \quad E_k = E_p - E_s
\]

- In general, \( C_{sat} (RH(t), T(t)) \) is both time- and temperature-dependent.
- Therefore, ANSYS cannot solve the problem with varying RH as function of time.

\[
E_p = 4.01 \times 10^4 \left( \frac{J}{mol} \right) = 0.415 \ (ev)
\]

\[
p_0 = 3.82 \times 10^{10} \ (Pa).
\]

\[
p_0 = 3.82 \times 10^{10} \ (Pa).
\]

\[
E_p = 4.01 \times 10^4 \left( \frac{J}{mol} \right) = 0.415 \ (ev)
\]
New Normalization Theory - $\tilde{C}_K$ Approach

- $\tilde{C}_K = \frac{C}{K}$ is used as field variable.
- $K(T)$: generalized solubility (kg/m$^3$)

$$\frac{\partial (K\tilde{C}_k)}{\partial t} = \nabla \cdot (D \nabla (K\tilde{C}_k))$$

$$\Rightarrow k \frac{\partial \tilde{C}_k}{\partial t} + \tilde{C}_k \frac{\partial K}{\partial T} \frac{\partial T}{\partial t} = \nabla \cdot \left([D]K \nabla \tilde{C}_k + \tilde{C}_k \frac{\partial K}{\partial T} \nabla T\right)$$

- The governing equation is exactly identical to the diffusion equation in ANSYS.
- $\tilde{C}_K$ turns out to be the water activity $a_w$ (Chen et al.), which has been proved to be continuous at interface.
Analogy using ANSYS

- Field variable and material property input

<table>
<thead>
<tr>
<th>Field Variable</th>
<th>Material Property</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{C}$</td>
<td>$C_{\text{sat}}$</td>
</tr>
<tr>
<td>$\bar{C}_k$</td>
<td>$K$</td>
</tr>
</tbody>
</table>

- Boundary condition

<table>
<thead>
<tr>
<th></th>
<th>$\bar{C}$</th>
<th>$\bar{C}_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Absorption</td>
<td>1</td>
<td>RH</td>
</tr>
<tr>
<td>Desorption</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

- Initial condition

<table>
<thead>
<tr>
<th></th>
<th>$\bar{C}$</th>
<th>$\bar{C}_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Absorption</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Desorption</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

- In using ANSYS,
  - $\bar{C}$ is replaced by $\bar{C}_k$.
  - Temperature-dependent material property $C_{\text{sat}}$ is replaced by general solubility $K$.
  - Boundary condition is now changed to $\bar{C}_k = \text{RH}$.

$$K = \frac{C_{\text{sat}}}{\text{RH}}$$

$$\frac{C_{\text{sat}}}{K} = \text{RH}(t)$$
Element Options in ANSYS

- **Two different types of element option**
  - **Diffusion element**
    - Plane 238 (2D, Quadratic)
    - Solid 239 (3D, Quadratic)
      - Degree of Freedom (DOF) label: CONC
  - **Coupled element**
    - Plane 223(2D, Quadratic)
    - Solid 226(3D, Quadratic)
      - Thermal-diffusion (100010)
        - DOF label: TEMP, CONC
      - Thermal-structural-diffusion (100011)
        - DOF label: TEMP, UX, UY, UZ, CONC
      - Structural-diffusion (100001)
        - DOF label: UX, UY, UZ, CONC

- Since the general diffusion equation in ANSYS involves with temperature derivative with respect to time when $K$ is temperature-dependent, the coupled element (thermal diffusion, or thermal-structural-diffusion) must be used.
Case Study – Varying RH and Temperature

- A 1-D bi-material absorption problem with varying RH and Temperature
  - Initial condition: dry at 30°C
  - Ambient temperature profile is given below to simulate an absorption process
    \[ T(t) \text{ [°C]} = 30 + \frac{50t}{200} \quad 0 < t \leq 200 \text{ (min)} \]
  - Relative humidity profile is given below to simulate an absorption process
    \[ RH(t) = 0.3 + \frac{0.7t}{200} \quad 0 < t \leq 200 \text{ (min)} \]
- Material properties

<table>
<thead>
<tr>
<th>Properties</th>
<th>Mat A</th>
<th>Mat B</th>
</tr>
</thead>
<tbody>
<tr>
<td>(D_0) (m²/s)</td>
<td>(5.0 \times 10^{-3})</td>
<td>(4.0 \times 10^{-3})</td>
</tr>
<tr>
<td>(E_D) (ev)</td>
<td>0.518</td>
<td>0.518</td>
</tr>
<tr>
<td>(S_0) (Kg/m/Pa)</td>
<td>(6.0 \times 10^{-10})</td>
<td>(2.0 \times 10^{-10})</td>
</tr>
<tr>
<td>(E_S) (ev)</td>
<td>0.383</td>
<td>0.362</td>
</tr>
</tbody>
</table>

- For this problem, \(C_{\text{sat}}\) is both time and temperature-dependent.
$C_{\text{sat}}$ as a Function of Temperature

- For the given material properties ($C_{\text{sat}}$ at 30%RH)
Finite Element Model and Results

- 1-D problem is constructed by a 2-D element strip model.
- Finite difference method (FDM) is used for verification.

- Coupled thermal-diffusion element presents correct results.

The starting point of the path $L_A = 1 \text{ mm}$

The end point of the path $L_B = 1 \text{ mm}$

$\text{h} = 5 \times 10^{-5} \text{ m}$

- Coupled thermal-diffusion element presents correct results.
\( \tilde{C} \) vs. \( \tilde{C}_k \) Approach

- \( \tilde{C} \) (coupled element with thermal-diffusion)

\[
C_{\text{sat}} \frac{\partial \tilde{C}}{\partial t} + \tilde{C} \frac{\partial C_{\text{sat}}}{\partial T} \frac{\partial T}{\partial t} + \tilde{C} \frac{\partial C_{\text{sat}}}{\partial t} = \nabla \cdot \left( [D] C_{\text{sat}} \nabla \tilde{C} + \tilde{C} \frac{\partial C_{\text{sat}}}{\partial T} \nabla T \right)
\]

- \( \tilde{C}_k \) (coupled element with thermal-diffusion)

\[
K \frac{\partial \tilde{C}_k}{\partial t} + \tilde{C}_k \frac{\partial K}{\partial T} \frac{\partial T}{\partial t} = \nabla \cdot \left( [D] K \nabla \tilde{C}_k + \tilde{C}_k \frac{\partial K}{\partial T} \nabla T \right)
\]

- Since \( C_{\text{sat}} \) is temperature-dependent and time-dependent, the results obtained by \( \tilde{C} \) are incorrect.
Comparison Among Different Elements in ANSYS

- Diffusion element only
  \[ K \frac{\partial \tilde{C}_k}{\partial t} = \nabla \cdot ([D] K \nabla \tilde{C}_k) \]

- Coupled element with structural-diffusion
  \[ K \frac{\partial \tilde{C}_k}{\partial t} = \nabla \cdot ([D] K \nabla \tilde{C}_k) \]

- Coupled element with thermal-diffusion
  \[ K \frac{\partial \tilde{C}_k}{\partial t} + \tilde{C}_k \frac{\partial K}{\partial T} \frac{\partial T}{\partial t} = \nabla \cdot \left( [D] K \nabla \tilde{C}_k + \tilde{C}_k \frac{\partial K}{\partial T} \nabla T \right) \]

- The Diffusion element only and the coupled element with structural-diffusion option present the same incorrect results.
Vapor Pressure Model - Example: Pressure Cooker

• Vapor phase

- Moisture in single vapor phase;
- \( \rho < \rho_g(T) \), \( \rho \): moisture density over the total volume of cooker; \( \rho_g(T) \): saturated moisture vapor density.
- Ideal gas law can be used: \( p_2 = p_1 \frac{T_2}{T_1} \)

• Liquid/vapor phase

- Moisture in two-phases – water/vapor mixed;
- \( \rho > \rho_g(T) \) - liquid-vapor phase, \( \rho \): moisture density over the total volume of cooker; \( \rho_g(T) \): saturated moisture vapor density
- Saturated vapor pressure remains regardless of water level
Vapor Pressure Model

- Moisture in free-volumes ‘free’ to vaporize
- Two distinct states
  - Single vapor phase
  - Mixed liquid/vapor phase
- Saturated moisture density $\rho_g(T)$
Vapor Pressure Model

$$\rho = \frac{dm}{dV_f} = \frac{dm}{dV} \frac{dV}{dV_f} = C / f$$

- $dV_f$: void volume in a REV (representative elementary volume)
- $dV$: total volume of a REV
- $dm$: total moisture mass
- $f$: void volume fraction $dV_f/dV$
- $\rho$: apparent moisture density

$$p(T) = \frac{RT}{MM_{H_2O} f} \cdot C$$

when $C(T) / f < \rho_g(T)$

$$p(T) = p_g(T),$$

when $C(T) / f \geq \rho_g(T)$


Steam Table - $p_g$

<table>
<thead>
<tr>
<th>$T(\degree C)$</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
<th>80</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_g (g/cm^2 \times 10^{-3})$</td>
<td>0.017</td>
<td>0.03</td>
<td>0.05</td>
<td>0.08</td>
<td>0.13</td>
<td>0.2</td>
<td>0.29</td>
</tr>
<tr>
<td>$p_g (MPa)$</td>
<td>0.002</td>
<td>0.004</td>
<td>0.007</td>
<td>0.01</td>
<td>0.02</td>
<td>0.03</td>
<td>0.05</td>
</tr>
<tr>
<td>$T(\degree C)$</td>
<td>90</td>
<td>100</td>
<td>110</td>
<td>120</td>
<td>130</td>
<td>140</td>
<td>150</td>
</tr>
<tr>
<td>$\rho_g (g/cm^2 \times 10^{-3})$</td>
<td>0.42</td>
<td>0.6</td>
<td>0.83</td>
<td>1.12</td>
<td>1.5</td>
<td>1.97</td>
<td>2.55</td>
</tr>
<tr>
<td>$p_g (MPa)$</td>
<td>0.07</td>
<td>0.1</td>
<td>0.14</td>
<td>0.2</td>
<td>0.27</td>
<td>0.36</td>
<td>0.48</td>
</tr>
<tr>
<td>$T(\degree C)$</td>
<td>160</td>
<td>170</td>
<td>180</td>
<td>190</td>
<td>200</td>
<td>210</td>
<td>220</td>
</tr>
<tr>
<td>$\rho_g (g/cm^2 \times 10^{-3})$</td>
<td>3.26</td>
<td>4.12</td>
<td>5.16</td>
<td>6.4</td>
<td>7.86</td>
<td>9.59</td>
<td>11.62</td>
</tr>
<tr>
<td>$p_g (MPa)$</td>
<td>0.62</td>
<td>0.79</td>
<td>1</td>
<td>1.26</td>
<td>1.55</td>
<td>1.91</td>
<td>2.32</td>
</tr>
<tr>
<td>$T(\degree C)$</td>
<td>230</td>
<td>240</td>
<td>250</td>
<td>260</td>
<td>270</td>
<td>280</td>
<td>290</td>
</tr>
<tr>
<td>$\rho_g (g/cm^2 \times 10^{-3})$</td>
<td>14</td>
<td>16.76</td>
<td>19.99</td>
<td>23.73</td>
<td>28.1</td>
<td>33.19</td>
<td>39.16</td>
</tr>
<tr>
<td>$p_g (MPa)$</td>
<td>2.8</td>
<td>3.35</td>
<td>3.98</td>
<td>4.69</td>
<td>5.51</td>
<td>6.42</td>
<td>7.45</td>
</tr>
</tbody>
</table>
Vapor Pressure: Effect of Reflow Profiles

Moisture diffusion modeling: a) SH, b) FM

Vapor pressure modeling: a) SH, b) FM
Vapor Pressure Modeling

CSP with thinner substrate

CSP with thicker substrate

– About 50% reduction on vapor pressure at 250°C in the bottom DA film between two thicknesses of substrate

Hygroscopic Swelling

Expansion strain due to hygroscopic swelling

Expansion strain due to temperature change of 100°C

$\varepsilon_{\text{hygro}} = \beta \times C$

$\beta$ – the coefficient of hygroscopic swelling

$C$ – moisture concentration

$\varepsilon_{\text{thermal}} = \alpha \times \Delta T$

$\sim 0.29\%$

$\sim 0.25\%$

Hygroscopic mismatch is comparable to thermal mismatch in causing mechanical stresses
Temperature-Moisture-Deformation Problems

- How to perform integrated modeling to accurately capture stress developments in each stage?
Material Properties to be Needed

\[
\varepsilon_{ij} = \frac{1 + \nu}{E} \sigma_{ij} - \frac{\nu}{E} \sigma_{kk} \delta_{ij} + (\alpha \Delta T + \beta C + \frac{1 - 2\nu}{E} p) \delta_{ij}
\]

- \(\alpha\): coefficient of thermal expansion (CTE)
- \(\beta\): coefficient of hygroscopic swelling (CHS)

C: moisture concentration
T: temperature, p: vapor pressure
Implementation in ANSYS

\[ \varepsilon_{\text{volu}} = \alpha \Delta T + \beta C + \frac{1-2\nu}{E} p \]

\[ \rightarrow \quad \varepsilon_{\text{volu}} = \alpha \Delta T + \beta \left( C + \frac{(1-2\nu)p}{\beta E} \right) \]

BF,,TEMP,n_temp  BF,,FLUE, b1+b2
Body Forces in ANSYS: Temp and Flue

- Total volume strain

\[ \varepsilon = \alpha (T-T_{\text{ref}}) + \beta C \]

\[ \text{BF,,TEMP,n\_temp} \quad \text{BF,,FLUE,n\_C} \]

ANSYS built-in swelling function

\[ \varepsilon^{sw} = \beta (\Delta C)^n \]

- \( \varepsilon^{sw} \): swelling strain
- \( \beta \): swelling coefficient
- \( C \): a fluence

TB,SWELL,1
! C72=10 FOR ACTIVATION OF USERSW
! EPS=C67*(FLUENCE)^C68 WHERE C67=SWELLING COEFFICIENT
TBDATA,72,10
TBDATA,67,\beta,1
Implementation in ANSYS

TB,SWell,MAT
TBDATA,72,10
TBDATA,67,β,1

ldread,temp,,,,moisture,rth
*do,i,1,total_node
*get,C,node,i,ntemp
BF,i,FLUE,C
*enddo

BF,,Temp,T_reflow

solve

• Moisture diffusion (including desorption) modeling needs to be solved first and separately.
  – ANSYS 17.2 or later provides coupled element
    ▪ Plane 223(2D, Quadratic)
    ▪ Solid 226(3D, Quadratic)
      ○ Thermal-structural-diffusion (100011)
        ✷ DOF label: TEMP, UX, UY, UZ, CONC
Example: Bi-Material Configuration

Moisture concentration contours

Deformed shape contours

von Mises stress
Contours in MC

Max stress = 4.96 MPa
Max stress = 20.99 MPa

Time = 1 hour @ 85°C/85%RH
Time = 18 hours @ 85°C/85%RH

Example: Reflow Profile Setting

- **Temperature (T):**
  - 85°C
  - 200°C
  - 260°C

- **Temperature Rates (°C/s):**
  - 0.5°C/s
  - 2°C/s

- **Time (t):**
  - 230s
  - 30s
  - 30s
Moisture Concentration and Vapor Pressure Contours

Moisture concentration contours

Max $C = 3.54 \mu g/mm^3$
$T=200^\circ C$

Max $C = 2.11 \mu g/mm^3$
$T=260^\circ C$

Max $C = 1.05 \mu g/mm^3$
$T=265^\circ C$

Vapor pressure contours

Max $p = 1.55 MPa$
$T=200^\circ C$

Max $p = 5.84 MPa$
$T=260^\circ C$

Max $p = 0.5e^{-3} MPa$
$T=265^\circ C$
## Results of Integrated Stress Modeling

### 200°C

<table>
<thead>
<tr>
<th></th>
<th>Von Mises Stress in Epoxy (MPa)</th>
<th>Von Mises Stress in Cu (Mpa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>2.4</td>
<td>8.7</td>
</tr>
<tr>
<td>T + H</td>
<td>4.9</td>
<td>17.3</td>
</tr>
<tr>
<td>T + H + V</td>
<td>6.3</td>
<td>22.9</td>
</tr>
</tbody>
</table>

### 260°C

<table>
<thead>
<tr>
<th></th>
<th>Von Mises Stress in Epoxy (MPa)</th>
<th>Von Mises Stress in Cu (Mpa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>6.0</td>
<td>21.9</td>
</tr>
<tr>
<td>T + H</td>
<td>7.2</td>
<td>27.5</td>
</tr>
<tr>
<td>T + H + V</td>
<td>11.2</td>
<td>38.9</td>
</tr>
</tbody>
</table>

### 260°C after 30sec hold

<table>
<thead>
<tr>
<th></th>
<th>Von Mises Stress in Epoxy (MPa)</th>
<th>Von Mises Stress in Cu (Mpa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>6.0</td>
<td>21.9</td>
</tr>
<tr>
<td>T + H</td>
<td>6.3</td>
<td>24.7</td>
</tr>
<tr>
<td>T + H + V</td>
<td>6.3</td>
<td>24.7</td>
</tr>
</tbody>
</table>
Summary

• For a general moisture diffusion problem with temperature-dependent $C_{\text{sat}}$ and varying ambient RH and temperature with time
  – $C_k$ must be used and the coupled element with thermal-diffusion or thermal-structural-diffusion option must be applied at the same time.

• ANSYS built-in $\bar{C}$ approach cannot solve the problem with varying RH correctly.
  – ANSYS diffusion element only
    ▪ $C_{\text{sat}}$ must be temperature-independent. Temperature gradient is not considered.
  – ANSYS coupled element with structural-diffusion option.
    ▪ $C_{\text{sat}}$ must be temperature-independent. Temperature gradient is not considered.
  – ANSYS coupled element with thermal-diffusion option (or thermal-structural-diffusion) option
    ▪ If $\bar{C}$ is used, RH must be constant.
    ▪ If $\bar{C}_k$ is used, no restriction for any diffusion problems.

• Vapor pressure model

• Thermal-hygro-mechanical modeling
• Introduction
• Temperature Loading
• Mechanical Loading
• Moisture and Humidity
• Electrical Current - Multi-Physics Modeling
• Summary
Outline

• Electrical-Thermal-Mechanical Modeling
• Electromigration
Electrical-Thermal Modeling

- Governing equations
  
  \[ \nabla^2 V = 0 \quad \iff \quad \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0 \]

  \[ k_T \nabla^2 T + \frac{1}{\rho} |\nabla V|^2 = 0 \quad \text{Joule heating} \quad |\vec{j} \cdot \nabla V| = \frac{1}{\rho} |\nabla V|^2 \]

- Current flow generates joule heating at any location.
- Joule heat is applied as heat source in heat diffusion in solids.
- Joule heat term:
  \[ q_0 = \frac{1}{\rho} |\nabla V|^2 \]

- We solve this multi-physics problem one step at a time
  - Both problems are diffusion-type
  - Two problems are sequentially coupled if electric conductivity does not vary with temperature
Example: A Microresistor Beam in MEMS

- Current flow modeling

COMSOL Multi-Physics Modeling Tutorial.
Thermal Modeling

Inside the material:
Thermal flux balance with the electric heating as source:
\[ Q = \sigma |\nabla \langle V \rangle |^2 \]

Convection conditions
flux out:
\[ h \times (T - T_{amb}) \]

Fixed temperature \( T_0 \)

Maximum temperature
Thermal-Mechanical Modeling

- Governing equations

\[ \nabla \cdot k[\nabla T] = c_p \rho \frac{\partial T}{\partial t} \]

\[ G \nabla^2 u_i + (\lambda + G)e_{,i} - \left[ \frac{E}{1-2\nu} \alpha T \right]_{,j} + X_i = 0 \]

- Temperature load is applied in the form of ‘body force’ in mechanical analysis

\( u_i \): the component of displacement vector
\( e \): the total volumetric strain \( e = u_{i,i} \)
\( X_i \): the component of body force vector.
Electrical-Thermal-Mechanical Modeling

- MEMS: microresistor beam modeling

  - MEMS actuator application
  - Electrical current
    - DC balance for conductive media
    - Fixed potentials generate potential difference $\Delta V = 2V$
  - Heat transfer
    - Thermal flux balance with the resistive heating as source
  - Structural analysis
    - Force balance with the thermally induced stress as volume load
Mechanical Modeling

- Results, Deformation

![Diagram showing force balance with thermally induced stress and deformation](image)

- Fixed to the base plate
What is Electromigration (EM)?

- Electromigration is a process of mass transport in the current-carrying metal under the driving forces generated by electric field.
- Electromigration is the most persistent reliability problem in interconnect technology in semiconductor device.

K.N. Tu: EPTC 2008 short course

IEEE EPTC 2019 – Singapore

Xuejun Fan and Ricky Lee

December 4 – 6, 2019
Similar to River Flow...
Driving Forces

- The atomic transport is caused by a combination of interacting driving forces that can generate voids at different locations.
Blech’s Theory (1976)

- EM flux is entirely balanced by the stress-induced counter flux.
- Blech Product, \( jL \), provides a threshold condition of maximum stress, below which, electromigration failure will not occur - mechanical failure.

\[ Z^*e\rho j + \Omega \frac{\partial \sigma}{\partial x} = 0 \]

\[ jL = \frac{(\sigma_{\text{max}} - \sigma_{\text{min}})\Omega}{Z^*e\rho} \]
Electromigration – a Coupled, Multi-Physics Problem

Total flux in terms of $C_v$ (vacancy concentration)

$$J_v = \frac{D_v C_v}{k_B T} (Z^* e \rho j - \frac{k_B T}{C_v} \nabla C_v - f \Omega \nabla \sigma + \frac{Q^*}{T} \nabla T)$$

- $C_v$: vacancy concentration (m$^{-3}$)
- $D_v$: diffusivity
- $k_B$: Boltzmann’s constant (J/K)
- $Z^*$: effective charge number (>0)
- $f$: volume relaxation ratio
- $\rho$: density
- $j$: current density (A/m$^2$)
- $\sigma$: stress
- $\Omega$: volume of per atom (m$^3$)
- $e$: elementary charge (C)
- $Q^*$: heat of transport (kJ/mol)

- EM is a multiphysics coupled field problem involved with electron wind, chemical potential, stress gradient and temperature gradient.
### An Overview in Literature

<table>
<thead>
<tr>
<th></th>
<th>Flux by self-diffusion</th>
<th>Flux by stress</th>
<th>Source/sink term</th>
<th>Constraint condition</th>
<th>Stress equilibrium</th>
<th>EM strain in stress/strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shatzkes and Lloyd (1986)</td>
<td>✓</td>
<td>×</td>
<td>×</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
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<tr>
<td>Kirchheim (1992)</td>
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<td>×</td>
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<tr>
<td>Korhonen et al. (1993)</td>
<td>×</td>
<td>✓</td>
<td>✓</td>
<td>×</td>
<td>×</td>
<td>×</td>
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<tr>
<td>Clement and Thompson (1995)</td>
<td>×</td>
<td>✓</td>
<td>✓</td>
<td>×</td>
<td>×</td>
<td>×</td>
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<tr>
<td>Sarychev et al. (2000)</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
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<td>×</td>
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<tr>
<td>Suo et al. (2003, 2011, 2014)</td>
<td>×</td>
<td>✓</td>
<td>✓</td>
<td>×</td>
<td>×</td>
<td>×</td>
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<tr>
<td>Sukharev et al. (2004, 2007)</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>N/A</td>
<td>✓</td>
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<tr>
<td>Maniatty et al. (2016)</td>
<td>×</td>
<td>✓</td>
<td>✓</td>
<td>×</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

• Inconsistent and incomplete solutions appear in literature.
New Theory – General Coupling Model

<table>
<thead>
<tr>
<th></th>
<th>Flux by self-diffusion</th>
<th>Flux by stress</th>
<th>Source/sink term</th>
<th>Constraint condition</th>
<th>Stress equilibrium</th>
<th>EM strain in stress/strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cui et al. (2019)</td>
<td>✓</td>
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<td>✓</td>
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<td>✓</td>
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</tr>
</tbody>
</table>

General coupling model for electromigration and one-dimensional numerical solutions

Cite as: J. Appl. Phys. 125, 105101 (2019); https://doi.org/10.1063/1.5065376
Submitted: 09 October 2018. Accepted: 19 February 2019. Published Online: 08 March 2019

Zhen Cui, Xuejun Fan, and Guoqi Zhang
Puzzle One

• Volume Strain in Confinement

**Literature**

\[ \theta = 0 \] (volume strain equals to zero) anywhere.

**Present solution**

\[ u(L) = u(0) = 0 \rightarrow \int_0^L \varepsilon_x dx = 0 \rightarrow \int_0^L \theta dx = 0 \]

**Confined conductor configuration**

• Equilibrium \( \left( \frac{d\sigma_x}{dx} = 0 \right) \) is violated in the existing literature theory and solutions.
Puzzle Two

- **Hydrostatic Stress $\sigma$ vs. Vacancy Concentration $C_v$**

**Literature**

\[
\sigma = \frac{k_B T}{\Omega} \ln\left(\frac{C_v}{C_{v0}}\right)
\]

**Present solution**

\[
\sigma = \frac{2EA}{9(1-v)} \left[ \frac{1 + v}{2(1-2v)L} \int_0^L \ln \left( \frac{C_v}{C_{v0}} \right) dx \right] + \ln \left( \frac{C_v}{C_{v0}} \right)
\]

Confined configuration

C. Herring, J. Appl. Phys. 21, 437 (1950)

- With the exact $\sigma$-$C_v$ equation, the stress level is significantly lower.
Puzzle Three

• Effect of Self-Diffusion

**Literature**

\[ j_v = -\frac{D_v C_v}{k_B T} (f \Omega \frac{\partial \sigma}{\partial x} + Z^* \rho j) \]

\[ J_v = \frac{D_v C_v}{k_B T} (Z^* \rho j - \frac{k_B T}{C_v} \nabla C_v - f \Omega \nabla \sigma + Q^* \frac{1}{T} \nabla T) \]

**Present solution**

\[ j_v = -\frac{D_v C_v}{k_B T} \left( \frac{k_B T}{C_v} \frac{\partial C_v}{\partial x} + f \Omega \frac{\partial \sigma}{\partial x} + Z^* \rho j \right) \]

\[ \sigma(0, \delta, \text{MPa}) \]

- When concentration gradient is considered, the stress becomes further lower.
Puzzle Four

• Stress-Strain Relation (Constitutive Equation)

**Literature**

Electromigration-induced volume strain

\[ \varepsilon_{kk}^{EM} = -\frac{\sigma}{B} \]

\[ \sigma = 2G\varepsilon + \lambda \text{tr}(\varepsilon)I - B\text{tr}(\varepsilon^T)I \]

\[ \varepsilon = \varepsilon^M + \varepsilon^T + \varepsilon^{EM} = 0 \]

**Present solution**

\[ \varepsilon_{kk}^{EM} = g(C_v) \]

\[ g(C_v) = -A \ln \left( \frac{C_v}{C_{v0}} \right), \quad A = \frac{k_B T}{B \Omega} \]

\[ \varepsilon = \varepsilon^M + \varepsilon^T + \varepsilon^{EM} \]

\[ \varepsilon^T = \alpha \Delta T I, \quad \varepsilon^{EM} = \frac{g(C_v)}{3} I, \quad g(C_v) = -A \ln \left( \frac{C_v}{C_{v0}} \right) \]

\[ \sigma = 2G\varepsilon + \lambda \text{tr}(\varepsilon)I - B\text{tr}(\varepsilon^T)I - B\text{tr}(\varepsilon^{EM})I \]

• A 3-D, general and self-consistent stress-strain constitutive equation is obtained.


### Puzzle Five

#### Atomic Transport Equation

**Literature**

\[
\frac{\partial C_v}{\partial t} + \nabla \cdot J_v = G \quad \text{(sink/source term)}
\]

\[
G = \frac{\partial C}{\partial t}
\]

\[
\frac{\partial C}{C} = d\varepsilon_{k\ell}^{EM}
\]

with approximation

\[
\frac{\partial C_v}{\partial t} \ll \frac{\partial C}{\partial t}
\]

\[
\frac{\partial \varepsilon_{k\ell}^{EM}}{\partial t} = \Omega \nabla \cdot J_v
\]

**Present solution**

\[
\frac{\partial \theta}{\partial t} = \Omega \nabla \cdot J_v
\]

\[
\theta = \varepsilon_{k\ell}^{M} + \varepsilon_{k\ell}^{T} + \varepsilon_{k\ell}^{EM}
\]

without any approximation

\[
\frac{\partial \varepsilon_{k\ell}^{M}}{\partial t} + \frac{\partial \varepsilon_{k\ell}^{T}}{\partial t} + \frac{\partial \varepsilon_{k\ell}^{EM}}{\partial t} = \Omega \nabla \cdot J_v
\]

- Mass conservation equation is used to describe the atomic transport. Thus, the sink/source term is considered naturally.
New Theory – General Coupling Model

Mass conservation equation:
\[
\frac{\partial \theta}{\partial t} = \Omega \nabla \cdot J_v \\
J_v = -D_v \nabla C_v + D_v C_v \frac{Z^* \rho j}{k_B T} - D_v C_v \frac{\Omega}{k_B T} \nabla \sigma + D_v C_v \frac{Q^*}{k_B T} \nabla T
\]

Constitutive equation:
\[
\theta = \text{tr}(\varepsilon), \quad \varepsilon = \varepsilon^M + \varepsilon^T + \varepsilon^{EM} \\
\varepsilon^T = \alpha \Delta T I, \quad \varepsilon^{EM} = \frac{g(C_v)}{3} I, \quad g(C_v) = -A \ln \left( \frac{C_v}{C_{v0}} \right) \\
\sigma = 2G\varepsilon + \lambda \text{tr}(\varepsilon) I - B\text{tr}(\varepsilon^T) I - B\text{tr}(\varepsilon^{EM}) I \\
\sigma = \text{tr}(\frac{\sigma}{3})
\]

Field equations:
\[
\nabla \cdot \sigma + F = 0, \quad \varepsilon = \frac{1}{2} (\nabla u + u \nabla) \\
\nabla \cdot j = 0, \quad j = \frac{E}{\rho} = -\frac{\nabla V}{\rho} \\
k\nabla^2 T + j \cdot E = 0
\]

Cui Z, et al., JAP, 125, 2019
Totally fixed configuration – 1-D problem

- **Assumptions**
  - 1-D problem: $\varepsilon_y = \varepsilon_z = \varepsilon_{xy} = \varepsilon_{yz} = \varepsilon_{xz} = 0$
  - Current density $j$ is constant and in negative $x$-axis direction.
  - Temperature is constant – no temperature gradient.
  - Perfectly blocking condition for $C_v$: $J_v(0,t) = J_v(L,t) = 0$.
  - Mechanically totally fixed: $u(L,t) = u(0,t) = 0$
1-D Governing Equations

\[
\frac{\partial \theta}{\partial t} + D_v \Omega \left[ \frac{\partial}{\partial x} \left( \frac{\partial C_v}{\partial x} + \frac{f \Omega C_v \partial \sigma}{k_B T} \partial x + \frac{Z^* \rho j C_v}{k_B T} \right) \right] = 0
\]

\[
\theta = -A \ln \left( \frac{C_v}{C_{v0}} \right) + \frac{3(1 - 2v) \sigma}{E}
\]

\[
\sigma = \frac{2EA}{9(1 - v)} \ln \left( \frac{C_v}{C_{v0}} \right) + \frac{1 + v}{3(1 - v)} \sigma_x
\]

\[
\sigma_x = \frac{(1 - v)E}{(1 + v)(1 - 2v)} \varepsilon_x + \frac{EA}{3(1 - 2v)} \ln \left( \frac{C_v}{C_{v0}} \right)
\]

\[
\frac{d \sigma_x}{dx} = 0
\]

\[
\int_0^L \varepsilon_x dx = 0 \quad u(L, t) = u(0, t) = 0 \rightarrow \int_0^L \varepsilon_x dx = 0 \rightarrow \int_0^L \theta dx = 0
\]

\[
\frac{\partial C_v}{\partial t} - \frac{1}{L} \int_0^L \frac{\partial C_v}{\partial t} dx = \frac{3(1 - v)D_v C_v \Omega}{(1 + v)A} \left[ \left( 1 + \frac{2EA \Omega}{9(1 - v)k_B T} \right) \frac{\partial^2 C_v}{\partial x^2} + \frac{Z^* \rho j}{k_B T} \frac{\partial C_v}{\partial x} \right]
\]

\[
\sigma = \frac{2EA}{9(1 - v)} \left[ \frac{1 + v}{2(1 - 2v)L} \int_0^L \ln \left( \frac{C_v}{C_{v0}} \right) dx + \ln \left( \frac{C_v}{C_{v0}} \right) \right]
\]
## Parameters and Physical Properties

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of interconnect ($L$)</td>
<td>50 $\mu$m</td>
</tr>
<tr>
<td>Young's modulus ($E$)</td>
<td>$70 \times 10^9$ Pa</td>
</tr>
<tr>
<td>Temperature ($T$)</td>
<td>500K</td>
</tr>
<tr>
<td>Poisson ratio ($\nu$)</td>
<td>0.3</td>
</tr>
<tr>
<td>Atomic diffusivity ($D_a$)</td>
<td>$3 \times 10^{-16}$ m$^2$/s</td>
</tr>
<tr>
<td>Vacancy diffusivity ($D_v$)</td>
<td>$3 \times 10^{-9}$ m$^2$/s</td>
</tr>
<tr>
<td>Atomic volume ($\Omega$)</td>
<td>$1.66 \times 10^{-29}$ m$^3$</td>
</tr>
<tr>
<td>Initial vacancy concentration ($C_{v0}$)</td>
<td>$6.02 \times 10^{21}$ m$^{-3}$</td>
</tr>
<tr>
<td>Atomic volume relation ($D_v C_v = D_a C_a$, $C_a = 1/\Omega$)</td>
<td></td>
</tr>
<tr>
<td>Electrical resistivity ($\rho$)</td>
<td>$4.88 \times 10^{-8}$ Ohm$\cdot$m</td>
</tr>
<tr>
<td>Current density ($j$)</td>
<td>$10^{10}$ A/m$^2$</td>
</tr>
<tr>
<td>Elementary charge ($e$)</td>
<td>$1.60 \times 10^{-19}$ C</td>
</tr>
<tr>
<td>Effective charge number ($Z^*$)</td>
<td>3.5</td>
</tr>
<tr>
<td>Boltzmann constant ($k_B$)</td>
<td>$1.38 \times 10^{-23}$ J/K</td>
</tr>
<tr>
<td>Coefficient of electromigration strain ($A$)</td>
<td>0.0071</td>
</tr>
</tbody>
</table>

Cui Z, et al., JAP, 125, 2019
Numerical Results ($\sigma$)

- The present results show significant differences with the previous results, particularly in the magnitude of hydrostatic stress. (113 MPa vs. 350 MPa: 30% of the stress value obtained).
Numerical Results ($C_v$)

- Vacancy concentration is comparable with the previous results, even though stress is significantly lower.
Effect of Self-Diffusion

- Normalized flux term,
  \[ \beta_1 = \frac{F_E(x,t)}{Z^*\rho j}, \quad \beta_2 = \frac{F_s(x,t)}{Z^*\rho j}, \quad \beta_3 = \frac{F_c(x,t)}{Z^*\rho j} \]
  where
  \[ F_E = Z^*\rho j, \quad F_s = -\Omega \frac{\partial \sigma}{\partial x}, \quad F_c = -\frac{k_B T}{c_v} \frac{\partial c_v}{\partial x} \]

- Self-diffusion plays more important role than stress in electromigration.
Numerical Results: Stress-Free Condition

- Governing equations,

\[
\frac{\partial C_v}{\partial t} = \frac{D_v C_v \Omega}{A} \left[ \frac{\partial^2 C_v}{\partial x^2} + \frac{Z* \rho_j}{k_B T} \frac{\partial C_v}{\partial x} \right]
\]

\[\sigma = 0\]

- Electromigration happens on the same time-scale.
- Electromigration is resisted by the counter force due to concentration gradient.
Stress-Free Condition (Lloyd’s Results)

• Governing equations

\[
\frac{\partial C_v}{\partial t} = D_v \left[ \frac{\partial^2 C_v}{\partial x^2} + \frac{Z^* e \rho j}{k_B T} \frac{\partial C_v}{\partial x} \right]
\]

• Electromigration happens in milliseconds.

• \(10^7\) orders difference compared to the confined configuration.

Revisit – Blech’s Theory

Blech’s theory (1976)

Cui, Fan, Zhang (2019)

• The effect of thermomigration is not taken into the consideration for the purpose of comparison.
Blech Theory’s Extension

• Confined Configuration

\[
(1 + \frac{2EAf\Omega}{9(1 - v)k_BT}) \frac{\partial^2 C_v}{\partial x^2} + \frac{Z^*e\rho j}{k_BT} \frac{\partial C_v}{\partial x} = 0
\]

\[
jL_B = \frac{2EAf\Omega + 9(1 - v)k_BT}{9(1 - v)Z^*e\rho} \ln \left( \frac{C_v,\text{max}}{C_v,\text{min}} \right)
\]

• Stress-free Configuration

\[
\frac{\partial^2 C_v}{\partial x^2} + \frac{Z^*e\rho j}{k_BT} \frac{\partial C_v}{\partial x} = 0
\]

\[
jL_B = \frac{k_BT}{Z^*e\rho} \ln \left( \frac{C_v,\text{max}}{C_v,\text{min}} \right)
\]

• The product of jL, is obtained for both confined and stress-free conditions.

• The vacancy concentration is used as threshold condition for failure criterion.
Threshold Product Curve

- The confined metal line can sustain the higher current density than that of stress-free condition.
- The experimental results are consistent with the present theoretical predictions.
  - In Blech’s experiment, the measured threshold product for the uncovered metal line is lower than that for the confined metal line.

I. A. Blech, JAP, 47, 1203, 1976.
Summary

- Electrical-thermal-mechanical modeling – sequential approach
- Electromigration
  - A 3-D general coupling model for electromigration is developed.
  - 1D solution for the confined configuration is obtained.
  - Mechanical stress-based failure criterion may not be valid anymore.
  - Blech’s theory is revisited and reanalyzed.
  - Vacancy concentration, instead of hydrostatic stress, used for failure criterion – a departure from the original view.
  - The new predictions are consistent with experimental observations.
• Introduction
• Temperature Loading
• Mechanical Loading
• Moisture Loading
• Electrical Current Loading - Multi-Physics Modeling
• Summary
Module 1 – Temperature Loading

• Thermal mismatch vs. temperature gradient

• Analytical solution
  – Layered structure (stress, warpage, effective CTE)
  – Cylindrical structure (TSV)

• Die-level thermal stress – thermal stress in TSV

• Package-level thermal stress problem – warpage

• Chip-package interaction (CPI) – submodeling technique

• Board level thermal stress problem
  – Solder ball thermal cycling (Flip chip BGA, WLP)
  – Creep equations
  – Best method for practice
    ▪ Initial stress free condition; full model vs. global/local model; worst solder ball location, volume averaging

• Stress singularity of joint materials
Module 2 – Mechanical Loading

- **JEDEC drop test standard**
  - JESD22-B111, old one, with 15 components
  - JESD22-B111A, new one, with 4 components or 1 component

- **Finite element modeling**
  - Input G method, large mass method, input displacement method, direct acceleration method
  - Global/local modeling
  - Peel stress used as indicator for failure

- **Four-point bending test and modeling**
  - Global/local modeling
  - Global with linear elastic but nonlinear geometry analysis
  - Local model with elastic-plastic modeling
Module 3 – Moisture Diffusion and Vapor Pressure

• For a general moisture diffusion problem with temperature-dependent $C_{sat}$ and varying ambient RH and temperature with time
  – $C_k$ must be used and the coupled element with thermal-diffusion or thermal-structural-diffusion option must be applied at the same time.

• ANSYS built-in $\bar{C}$ approach cannot solve the problem with varying RH correctly.
  – ANSYS diffusion element only
    ▪ $C_{sat}$ must be temperature-independent. Temperature gradient is not considered.
  – ANSYS coupled element with structural-diffusion option.
    ▪ $C_{sat}$ must be temperature-independent. Temperature gradient is not considered.
  – ANSYS coupled element with thermal-diffusion option (or thermal-structural-diffusion) option
    ▪ If $\bar{C}$ is used, RH must be constant.
    ▪ If $\bar{C}_k$ is used, no restriction for any diffusion problems.

• Vapor pressure model.

• Thermal-hygro-mechanical modeling.
Module 4 – Electrical Current Loading

• Electrical-thermal-mechanical modeling – sequential approach

• Electromigration
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Thank you for your attention.

Questions?