Introduction to Structural Steel Design

Steel is made out of 98% iron, 0.15 to 1.7% carbon, and other elements such as silicone, manganese, and sulfur and phosphorous. If the percentage of the other elements mentioned is high then steel is known as alloy steel. The more carbon the steel contains, the more brittle it becomes with higher strength. Since ductility is the property we are most interested in, the percentage of carbon is usually under 0.5.

Advantages of Steel:

a) High strength per unit weight especially when compared to concrete. This can reduce the size of the elements in the structure and increase the living space.
b) Uniformity: that reduces the effect of time on steel, as compared to concrete that changes through out its life.
c) Elasticity: steel is elastic, that is it follows Hook's Law as long as its stresses do not exceed its yielding stress.
d) Moment of inertia of steel is accurately calculated where as that of concrete changes as the cracks move up towards the neutral axis and past it.
e) Permanence: the better the maintenance the longer is its life. With modern steel, it can be rolled to satisfy its purpose with little maintenance required.
f) Ductility: since steel is a ductile material, it can undergo extensive deformations after which increased stresses are required for failure to occur. This is a property that can save lives.
g) Fracture toughness: toughness is the ability of the material to absorb energy. Since steel has to be transported and then erected, it will be exposed to various types of sudden stresses (drilled, punched, hammered, banged around ...) that it should be able to absorb without large strength reduction.
h) It is easier to add to a steel structure than it is to a concrete structure mainly due to connections.
i) It is faster to build a steel structure than it is a concrete structure due to its lightness compared to concrete, it requires no curing time, and the members are easily connected (bolted, welded, and riveted).
j) Recycled steel is a big part of the industry today. This allows the manufacturing of steel with 50 ksi yielding strength at a similar cost of producing the commonly used 36 ksi.

Disadvantages of Steel:

a) Maintenance cost: steel requires maintenance against corrosion. However this cost can be eliminated by using atmospheric corrosion-resistant steels such as A242 and A588.
b) Fireproofing costs: steel will not ignite. However, at 1200°F steel has very little strength. Its temperature should not exceed 800°F beyond which its strength is reduced quickly.
c) Buckling: can occur when long slender steel members are exposed to compressive loads. To avoid buckling, a larger cross-section is needed which will increase cost.

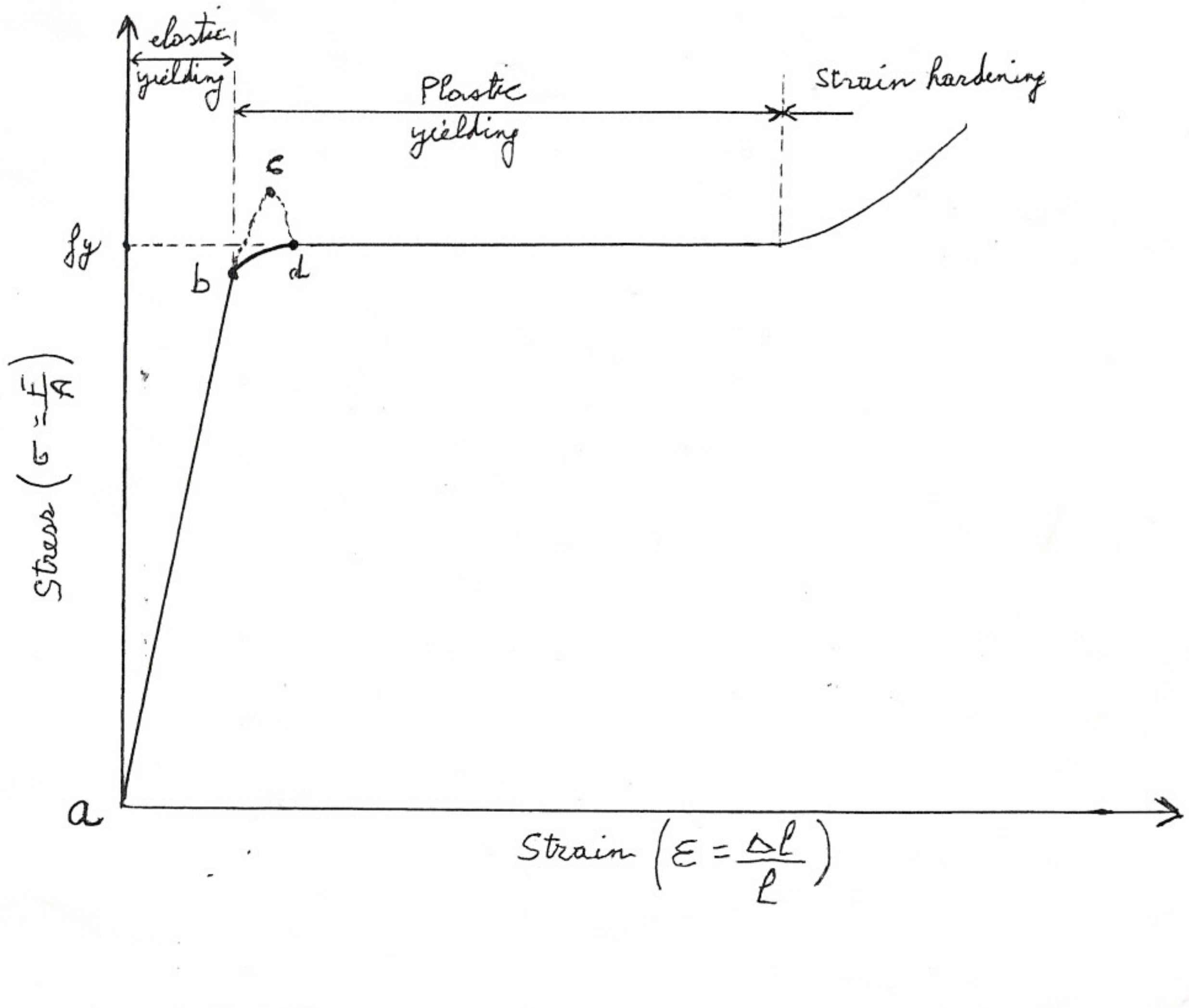
d) Fatigue: is caused by a large number of repetitive tensile stress variations. This can reduce the strength and ductility of the steel causing a sudden failure.

Steel Sections:

Besides having standardized shapes on the market, steel can be manufactured in any desired shape and cross-section.
The shapes or cross-sections with the highest moments of inertia are the most popular (I-beams, W sections, Tee shapes) because they can resist more bending stresses: $\sigma = Mc/I$ the higher the I the less the σ.
The LRFD manuals provide all the standardized shapes with all their characteristics such as dimensions, moments of inertia, radii f gyration, weight, ...
The LRFD manual uses an abbreviation system for all the shapes, few examples are:

> W27×114 represents a W section with a depth of 27 in and weight of 114lb/ft
>
> C10×30 represents a channel 10 in deep and weight of 30 lb/ft

Stress-Strain Relationships for Steel:



Referring to the above figure:

1) From a to b the elongation increases constantly with stress and Hook's Law applies.
2) The stress at point b is the final stress where Hook's Law applies (stress -strain relationship is not constant beyond that point). It is called the Proportional Limit, Engineers refer to it as the Elastic Limit, and it is also known as the Proportional Elastic Limit. Stresses beyond that point will cause permanent deformations.
3) Point c is the upper yield point (caused by rapid load application); point d is the lower yield point (caused by slow load application). The yield point is where the stress is constant while elongation increases.
4) The tangent to the curve at the yield point is a horizontal line.
5) At point b the stress equals half the ultimate strength of steel.
6) The shape of this curve can change with the type of loading, temperature, and type of steel.
7) Brittle steel can fail suddenly without any yielding. Its stress vs. strain diagram will be similar to that of concrete.

<u>Modern Structural Steel:</u>

With today's technology, the chemistry of steel can be altered to produce a variety of types of steel to fit almost every need. Refer to table 1-1 for the most common types produced.
The binding force between the iron atoms is estimated to be over 4000 ksi. This is why we have ultra high strength steels with yielding stresses up to 300 ksi (not in the LRFD yet) and the industry is experimenting with steels with up to 500 ksi yielding stress. Since the yield stress can be increased without a major cost increase, high strength steel can be soon used due to the following advantages:
a) High corrosion resistance
b) Light weight that reduces the cost of shipping and erection.
c) Smaller members will be required reducing the maintenance cost and increasing the living space.

<u>The Structural Designer:</u>

The first task a structural designer has, is to determine the loads the structure can be exposed to during its lifetime. Once the loads and their combinations are calculated, the designer has to find members that will sustain those loads. While doing that, the following points are to be taken into consideration:
1) Safety: the erection of the structure itself have to be accomplished in a safe way. Once that is done, the occupants should feel safe and hence should not see large cracks, deflections or sway.
2) Cost: low cost should be achieved without jeopardizing safety. Therefore, a low cost high strength structure should be designed by using standard size members, simple connections, and low maintenance ......

3) Practicality: will develop with experience. The more the designer is involved with the fabrication process, labor, erection, transportation, equipment used, the more practical, safe and cost efficient will his design be.

Design of Steel Members:

Designing is not just finding the lightest member that can support the specified load. The designer should consider the following while choosing his members:
1) Pick members that are rolled and are available in the market.
2) Use the same members within the same floor although lighter members may work.
3) Can the desired member be transported from the mill to the construction site?
4) Pick members that can accommodate the rest of the structure (pipes, electric, plumbing, AC systems)
5) Although it may cost a little more, nowadays-exposed structures are preferred to look good.

Calculation Accuracy:

The experience of the designer is what brings him to accurate calculations as humanely possible. However, the design starts by assuming the possible loads, then the analysis method is based on true assumptions and finally the strength of the material used can vary. Therefore, we can not claim design to be an accurate science, and that is why safety factors are needed.

Chapter 2

Specifications, Loads and Methods of Design

Specifications and Building Codes: are good guides to the designer, they secure structural safety, and protect the public. Municipalities and state governments develop the codes that engineers have to go by while designing. Most government agencies get their codes from organizations that develop specifications for guiding the designer. AISC and AASHTO are 2 examples.

Loads: To be able to design a safe, efficient and economical structure, we have to have an accurate idea of the types of loads the structure will be exposed to during its life time, and what combinations of these loads can occur at the same time.

Types of Loads:

1) Dead Loads: have a constant magnitude and a fixed position. That includes the structures own weight and anything fixed to it. However, to estimate the structures weight we have to know what members are being used. Therefore, we assume the members then check our results. The more experience the designer has, the lower the number of member estimates he has to do.
2) Live loads: change in magnitude and position. If it is not a dead load then it is a live load. Live loads are of 2 types: a) moving loads that move by their own power (cars and trucks). b) movable loads (furniture).
   Few examples of live loads are:
   a) floor loads: measured in lb/ft^2. Different types of structures have different floor load requirements. For example: 40 lb/ft^2 for appartements and 100 lb/ft^2 for office lobbies.
   b) Snow and ice: one inch of snow is equivalent to 0.5 lb/ft^2. Normal values range from 10 to 40 lb/ft^2 for flat to slopped roofs up to 45° angle.
   c) Rain especially on flat roofs because ponding develops causing deflections.
   d) Traffic loads for bridges.
   e) Impact loads: such as falling objects or sudden car braking.
   f) Lateral loads: such as wind, which changes with height, geographic location, surrounding structures… Wind loads should be designed for, if height of structure divided by the least lateral dimension is greater than 2. Wind acts like pressure and on a vertical surface can be estimated to be $P(lb/ft^2) = 0.002558 C_s V^2$ where C_s is a shape coefficient and V is wind velocity in miles/hour.

Earthquakes are another example of impact loads. They create seismic forces. This horizontal acceleration of the ground needs to be considered in design. The effect of earthquakes on buildings depends on the mass distribution of the buildings above the level being considered, and the ability of the soil to withstand the lateral motion.

g) longitudinal loads: such as sudden stopping of trains or trucks on bridges.
h) Other live loads: soil pressure on walls or foundations, water on dams, explosions, thermal forces due to temperature changes….

Selection of Design Loads: besides all the specifications and building codes available, an engineers experience and insight in the future helps him select design loads accurately.

Elastic Design: or allowable stress design or working stress design. In these cases the loads are estimated and the members designed according to their allowable stresses (a fraction of the minimum yield stress of steel). Method described in appendix A.

Plastic design: method estimates the loads and multiply them by a safety factor and members are designed based on collapse strength. Therefore the steel is used to its maximum limit making the approach more economical.

Load and Resistance Factor Design (LRFD): method is based on a limit state philosophy. Limit state: a structure or a part of the structure does not do its function. It is devided into two categories: 1) Strength limit state based on the structures load carrying capacity, buckling strength, plastic strength, fatigue and fracture. 2) Serviceability limit state based on how the structure act under normal loads, such as deflections, cracking, vibrations, and slipping.

In the LRFD method the working or service loads are multiplied by a safety factor, usually greater than 1, and the structure is designed to have an ultimate strength sufficient to support the factored load.

Ultimate strength = nominal strength or theoretical strength $\times \varnothing$

Where $\varnothing$ is less than one to account for possible uncertainties in design or materials.

Load Factors: increase the values of the loads to account for uncertainties. Since dead loads are estimated more accurately than live loads, their factors are smaller than those of live loads. The LRFD manual provides many load formulas for various load combinations. For example: the usual load combinations used are given by equations A4-1 and A4-2.

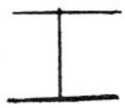
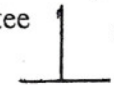
Resistance Factors: account for uncertainties in materials strength, dimensions and workmanship in determining the ultimate strength of the structure. Therefore the theoretical ultimate strength (nominal strength) is multiplied by ∅ (< 1).
for columns ∅ = 0.85 for tension members ∅ = 0.75 or 0.9
for bending or shear in beams ∅ = 0.9
Refer to table 2-2 page 57.

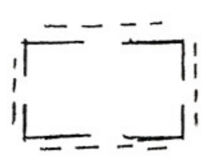
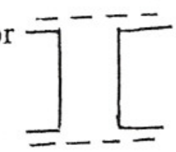
By using high load factors and small resistant factors we are protecting our design and the public from uncertainties that can occur due to material strength, method of analysis, forces of nature, stresses during construction and the production of the material, and the accuracy of the designed live load.

Reliability and the LRFD Specifications: Reliability is the estimated percent of times that the strength of the structure will equal or exceed the maximum loading applied to that structure over its life time. Most LRFD designs give 99.7 % reliable structures. That means 0.3 % of the times the structure will be overloaded and is pushed to its plastic limits or even to the strain hardening causing serious damage.

CHAPTER 3

ANALYSIS OF TENSION MEMBERS

Designing tension members is simple since they do not buckle. Knowing the load applied, and the stress limit of the material, we calculate the required area. Then from the LRFD manual we find a section that has at least that area. However, the LRFD offers a variety of sections such as angles  , W or S sections  , Structural tee  ,

or even built up sections from several single sections such as  , or 

The dotted lines are tie plates or tie bars that connect the members together giving them stiffness. If the connecting plates are perforated plates then they are considered as load carrying members.

<u>Design Strength of Tension Members:</u> a) Members with no holes drilled in them:

$\sigma = F/A$ where σ = stress A = cross-sectional area of member
F = load carried by the member

If steel is ductile, $F > A \times \sigma_{yielding}$ due to strain hardening. However, if the member is loaded to strain hardening, it will show a large increase in length and becomes useless and it can even fail causing the structure to fail.

b) If the member is connected by bolts and has bolt holes in it: in this case failure can occur in one of two ways: 1) Fracture failure at the net section through the holes. The load causing this failure can be less than the load required to yield the gross section away from the holes. 2) Yielding failure of the whole gross section.
Therefore, the design strength in this case is the smaller off:

I) Limit state of yielding in the gross section:

$P_n = F_y A_g$

and

$P_u = \phi_t F_y A_g$ where $\phi_t = 0.9$

II) Fracture in net section where holes are present:

$P_n = F_u A_e$

and

$P_u = \phi_t F_u A_e$ where $\phi_t = 0.75$

Where F_u = specified minimum tensile stress

Ae → A_e = effective net area that can be assumed to resist tension at section with holes in it.

Table 1-1 in LRFD manual provides values for F_u and F_y

Net Areas: Placing a hole in steel reduces the area of steel that carries loads in tension or compression, and hence increases the stress and their concentrations around the holes. However, if loaded beyond its yield stress, at its ultimate load we can assume the stress is uniformly distributed over the net area unless members will be exposed to fatigue loadings.

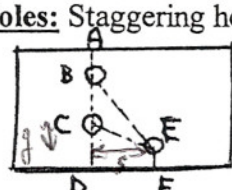
Net Area = Gross Area - Area of Holes, Notches or Indentations

Where Area of Holes = Hole Diameter × Steel Thickness

and Hole Diameter = Bolt Diameter + 1/8″

Note: Punched area is 1/8″ larger than the bolt diameter.

Effect of Staggered Holes: Staggering holes can increase the net area and hence increasing P_n and P_u. If holes are staggered every one of them and we can have different net areas. We find then the smallest one controls.



In case of staggering: The Net Area =

[gross width of the member - diameter of all holes along the considered line + s^2/4g] × thickness of plate.

If the cross-section has different thicknesses then :

The Net Area =

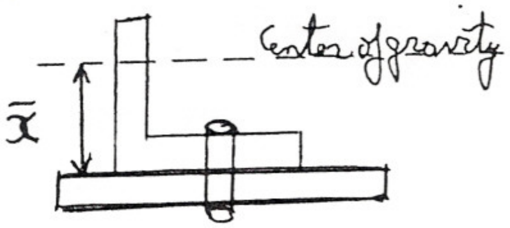
gross area of the member - diameter of holes × thickness + s^2/4g × thickness

Effective Net Areas: if a member's cross-section is made of more than one member (built-up section), then the forces are not transferred uniformly across its cross-section. Hence, the stresses at the connections and along a certain length of the member is higher and hence the cross-section will fail before the steel reaches its failure tensile stress. The uneven high stresses will extend from the connections to a certain length of the member, this is the transition region, before stresses are equally distributed again. The stresses in the transition region can exceed F_y, also shear lag can occur causing damage unless the load is reduced.

Effective Net Area = A_e = AU where A = gross area or the net area.

U = reduction factor to account for unequal stress distributions

Note: the smaller $\bar{x}$ the larger the value of A_e



Note: for bolted members A = net area of member

$U = 1 - \bar{x}/L \leq 0.9$ where x comes from manual

L = connection length (Length of line with maximum number of bolts)

Table 3-2 page 80 gives different values of U for different bolted conditions of different sections.

Note: for welded members $A_e = AU$ where A and U will have different values according to the situations given on page 80 of the book.

<u>Connecting Elements for Tension Members:</u> such as splices or gusset plates. The strength of these connections is:

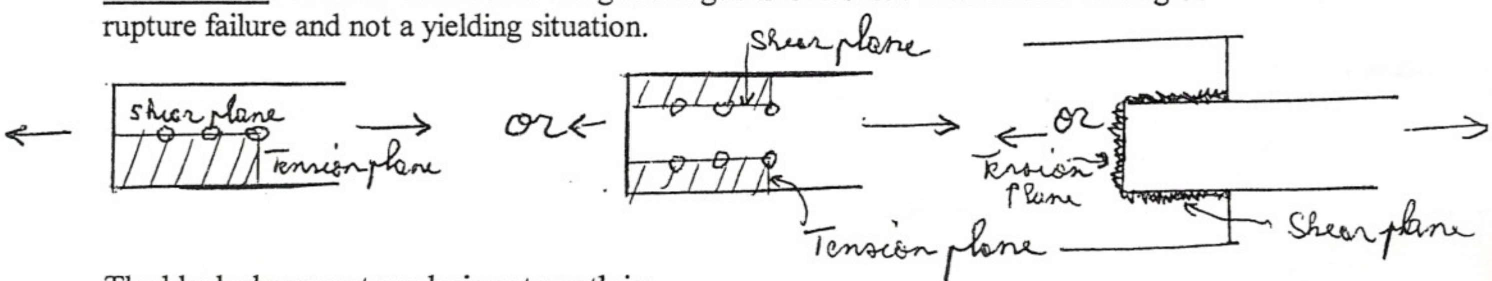
a) For yielding of welded or bolted connections: $R_n = A_gF_y$ and $\phi = 0.9$

b) For fracture of bolted connections: $R_n = A_nF_u$ and $\phi = 0.75$

Where $A_n \leq 0.85F_y$

and $A_n \leq 85\%A_g$

<u>Block Shear</u>: can also control the design strength. Block shear can create a tearing or rupture failure and not a yielding situation.



The block shear rupture design strength is:

1) If $F_uA_{nt} \geq 0.6F_uA_{nv}$ we will have shear yielding and tension fracture.

Therefore, $\phi R_n = \phi[0.6F_yA_{gv} + F_uA_{nt}]$

2) If $0.6F_uA_{nv} > F_uA_{nt}$ we will have tension yielding and shear fracture.

Therefore, $\phi R_n = \phi[0.6F_yA_{nv} + F_yA_{gt}]$

For both cases: $\phi = 0.75$

A_{gv} = gross area subject to shear

A_{gt} = gross area subject to tension

A_{nv} = net area subject to shear

A_{nt} = net area subject to tension

CHAPTER 4

DESIGN OF TENSION MEMBERS

Selection of Members: we have a given tension load, we need to find a member to support it. The member found should have: a) Compactness to give the member stiffness, b) Fit in its place in the structure and with other parts of the structure, c) Should be connected properly to the structure to prevent Shear Lag.
The type of connection we choose to use, will effect the type of section selected. Angles, W-sections and S-sections can be easily bolted, whereas plates, channels, and Tees are welded easier.

* Slenderness Ratio = Unsupported length/ Least Radius of Gyration.

The specifications give maximum allowable slenderness ratios to prevent vibrations and lateral deflections and to ensure stiffness. Tension members may even be exposed to buckling during shipping or earthquakes and hence the slenderness ratio is specified to prevent that by giving the tension member some compressive strength.

For tension members:
For members other than rods max S.R. = 300
The designers experience will decide what S.R. to use for rods because their radii of gyration is too small and hence their S.R. is usually too large (greater than 300)
For compression members:
Maximum S.R. = 200

DESIGN: If the tensile load P_u is given, we need to find the area of the cross-section. In this case we will have 2 areas to check for A_g (gross area) and A_e (effective area):

$$\text{Min } A_g = P_u/\phi_t F_y \quad \text{and} \quad \text{Min } A_e = P_u/\phi_t F_u$$

but for bolted members $A_e = A_n U$ Therefore Min A_n = Min A_e/U

Therefore Min $A_n = P_u/\phi_t F_u U$

Therefore the A_g for the 2nd formula should be: Min A_g = Min A_n + **Estimated** hole area (Estimated because we do not know the member yet and hence we do not know its thickness)

Therefore Min $A_g = P_u/\phi_t F_u U$ + **Estimated** hole area

Also $r_{min} = L/300$ which will not let the slenderness ratio exceed 300

Finally the largest A_g controls.

Step by step procedure:
1) Find the value of P_u using formulas for the different types of loading.
2) Find Min $A_g = P_u/\phi_t F_y$.

3) Using table 3-2 assume a value for U, and select a desired section that has a Min Ag that is close to the one found in step 2.
4) Find Min $A_g = P_u/\phi_t F_u U$ + **Estimated** hole area
(use the thickness of the member selected in step 3)
5) Find r = Min L/300
6) Use the largest A_g from the 2 calculated above, and r and find a section from the manual.
7) Check the section by doing the analysis like we did in chapter 3.
Briefly: Calculate $P_u = \phi t F_y A_g$ and $P_u = \phi t F_u A_e$ and they should both be greater than the given P_u (from the given loads).

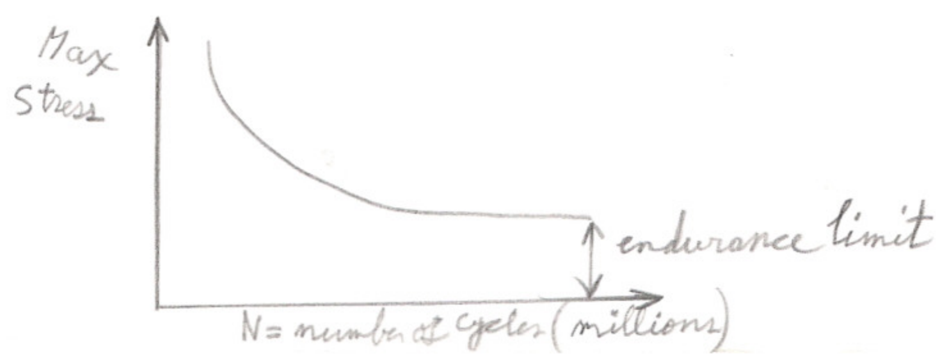
Built Up Tension Members: Section D2 and J3.5 of the LRFD Manual gives specific rules to connect tension members. The book on page 107 gives a few of those rules.

Rods and Bars: When used they can be welded or threaded and used with nuts.
For threaded rods $A_D = P_u / \phi\ 0.75F_u$ with $\phi = 0.75$
and $P_u > 10K$ except for lacing sag rods or girts.
Girts: are horizontal beams used on sides of industrial buildings to resist lateral bending due to wind.
Sag rods: provide support to purlins parallel to the roof surface and vertical support for girts along the walls.
The A_D of the rod is from the diameter of the outer extremity of the threads.
Table 8-7 in the manual gives properties of the standard threaded rods.
Threads reduce the area of the bars, this problem is fixed by "upsetting" the rods, that is making the threaded part larger than the actual rod. This allows us to use the entire cross-section of the rod in design calculations. However, upsetting is costly and hence it is usually avoided.

Pin Connected Members: Are not used a lot anymore because the pins in the holes wear out and the connection becomes loose. Example: Eye bar

Design for Fatigue Loads: Frequent stress variations or even reverse stresses can cause fatigue. For example if cranes or vibration causing machinery are used in the structure, we should consider fatigue. If subjected to tension stresses or stress variations, the member will form cracks that will spread and cause the structure to fail before the actual strength of the member is reached.
Note: Steel designers still do not have a perfect idea about fatigue failures although numerous tests have been conducted.

S (Maximum stress) vs. N(Number of cycles to failures in millions) curves are used to show fatigue failures and it varies with the grade of steel and temperature.



Since we have the endurance limit, The stress where the life seems to be infinite, we can design members that will not fail due to fatigue by making sure that the stress they will be exposed to does not exceed the endurance limit.

When do we design for stress fatigue?
If anticipated cycles > 20,000, a permisable stress range should be calculated. This is done as follows:

1) Determine the loading condition using table A-K3.1 of appendix K of the LRFD Manual (We have several loading conditions.)
2) Determine the type and location of material from figure A-K3.1 of the appendix.
3) Determine the stress category (A, B, C, D, E, or F) from table A-K3.2
4) Determine the allowable stress range for the service load and stress category from table A-K3.3
5) Find P_u according to the different conditions. For example, if exposed to reversal tension and then compression, find P_u for tension and then P_u for compression.
6) Then find A_g using the formulas discussed at the beginning of the chapter.
7) Check the design.

INTRODUCTION TO AXIALLY LOADED COMPRESSION MEMBERS

CHAPTER 5

Columns are an example of compression members that are widely used in construction. Columns are vertical members with a large length to thickness ratio. Short vertical members are also known as struts.

Types of failure for axially loaded columns:

1) Flexural buckling (Euler Buckling): members are subjected to flexure or bending when they become unstable.
2) Local buckling: a part or section of the column buckles before the rest of it does due to its small thickness at some parts of its cross-section.
3) Torsion buckling (chapter 6)

Buckling can occur due to:

1) Manufactory defects.
2) Eccentric load application (not at center of column)
3) Weak end connections.
4) Column not erected straight.

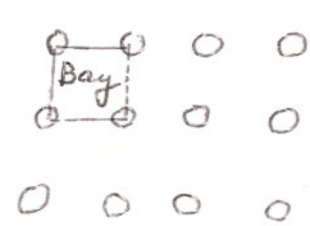
A columns tendency to buckle is measured by its slenderness ratio = L/r_{min}
where L = length of column
and r_{min} = least radius of gyration

A perfect column will have homogeneous materials throughout its length, will be erected perfectly straight, and loaded exactly at its center. However, because of the following a perfect column does not exist:

1) Cross-sectional imperfections.
2) Residual stresses.
3) Holes punched for bolts.
4) Erection stresses.
5) Transverse loads.

Those imperfections in columns can cause serious problems. The most important of these problems is developing bending moments for the columns that are not erected straight. Therefore, the design should take into consideration the stresses due to bending as well as axial loading (chapter 11).

Column spacing is important for economical purposes and planning.
The space 4 columns enclose is called a bay.



When shallow spread footings are used, Bays with length/width = 1.25 to 1.75, and with areas of approximately 1000 ft^2 will usually constitute an economical design.

Residual Stresses: are a result of:
1) Uneven cooling of shapes after hot rolling. Quick cooling resists further shortening developing residual compressive stresses, whereas slow cooling causes more shortening developing residual tensile stresses.
2) Welding 3) Cambering
Residual stresses can vary from 10 to 20 ksi and are to be considered especially for columns with slenderness ratios of 40 to 120. Columns with residual stresses reach their proportional limit at a stress equals to half of their yield stress and the stress versus strain relation is not linear from zero up to the yield point.
A column with residual stresses is like a column with reduced cross-sectional area.

Sections used for columns: depends on availability, type of connection, type of structure. (Refer to figure 5-2 for few examples). However, the best compression member has a constant radius of gyration about its centroid.

Development of column formulas: In 1757 Leonhard Euler realized the importance of buckling in columns.
1) For short columns failure stresses are close to yielding stresses.
2) For intermediate columns, tests showed that residual stresses usually control failure.
3) For long columns the end support conditions control failure.

The Euler Formula: The buckling stress decreases as the length of the column increase. At a certain length, and at lengths longer than that certain length, the buckling stress equals the proportional limit stress and we get an elastic buckling stress.
Euler found the load that causes this elastic buckling:

$$P = (\Pi^2)EI/(r^2)$$

From this equation we can see that the strength of the steel does not effect P.

In terms of slenderness ratio, the critical buckling stress will be:

$$\sigma = P/A = (\Pi^2)E/(L/r)^2$$

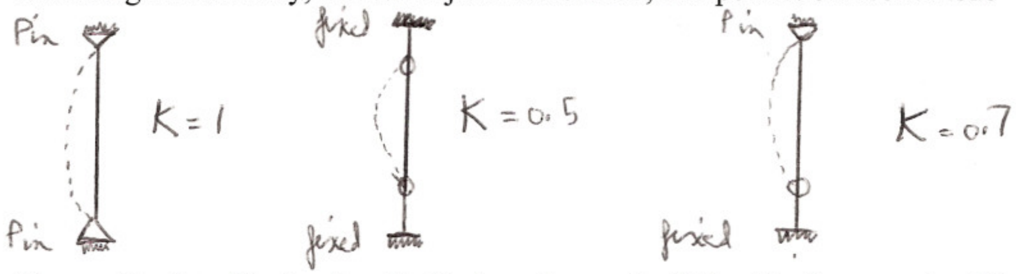
This value is expressed as F_e in the LRFD manual

If the critical buckling stress is greater than the proportional limit stress, then the column is not in its elastic buckling stage and Euler's Formula will not apply.

End restraints and effective lengths of columns: Euler's Formula gives best results if the end supports are considered and the effective lengths are used for L. The better the end restraints the better the load carrying capacity. The effective lengths are the distance between inflection points (points of Zero moments).

Effective length = KL where L = original column length
And K = effective length factor depending on end restraint and resistance to lateral movement.

Assuming no sidesway, no lateral joint translation, and perfect end conditions



The smaller the effective length, the less chance it will buckle, the more load it will carry. However, this perfection is practically impossible, and the values of K used in design are shown in table 5-1 page 141. These design values given for K are for preliminary designs and will not work for continuous columns (chapter 7).

Stiffened and unstiffened elements: Unstiffened elements have one free edge in the direction of compression, whereas stiffened elements are supported on both ends.
A part of a member can buckle before the buckling stress of the entire member is reached (ex: thin flange or web). To prevent this, the LRFD provides width to thickness ratio limits of the individual parts of a section. To do that members are classified as:
1) Compact sections: are stocky, buckling does not occur before the yield stress is reached, width to thickness ratio is less than λ_p (limiting width to thickness ratio) (Table 5-2).
2) Noncompact sections: yield stress is reached in some but not all of its compression elements before buckling, the width to thickness ratio is greater than λ_p but less than λ_r (table 5-2)
3) Slender compression elements: where the width to thickness ratio does not satisfy the conditions of table 5-2. However, the stress reduction is severe and it is not economical to use slender members.

Short, long and intermediate columns: the strength of a column and the way it fails highly depends on its effective length. As the effective length increase, the buckling stress decreases.
1) Long columns: buckling stress is less than the proportional limit stress. Therefore, elastic buckling controls and Euler's Formula applies.
2) Short columns: failure stress equals the yield stress. Therefore buckling does not occur. In actual design this does not exist.
3) Intermediate columns: some fibers yield and some do not. This is inelastic behavior, and failure is by yielding and buckling. To use Euler's Formula in this case we have to modify it and account for residual stresses. Most columns fall in this range.

Column Formulas: determine critical buckling stresses Fcr. From the critical buckling stress Fcr we can get:

P_n = Nominal Strength = A_gF_{cr}

and P_u = Ultimate Strength = $\varnothing A_gF_{cr}$ where $\varnothing = 0.85$

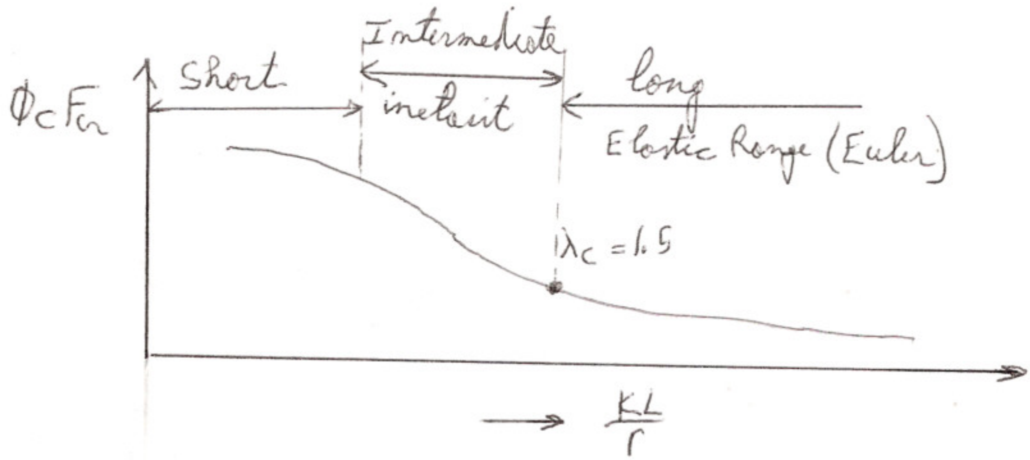
Considering residual stresses and out of straightness:

For inelastic buckling: $F_{cr} = [(0.658)\text{^}(\lambda_c)\text{^}2]F_y$ for $\lambda_c \leq 1.5$

For elastic buckling: $F_{cr} = [0.877/\lambda_c\text{^}2]F_y$ for $\lambda_c > 1.5$

Where $\lambda_c = (F_y/F_e)\text{^}0.5$ and $F_e = \Pi\text{^}2E/(KL/r)\text{^}2$

Therefore $\lambda_c = (KL/\Pi r)(F_y/E)\text{^}0.5$



However instead of using formulas, the LRFD manual provides values for $\varnothing_c F_{cr}$ for various KL/r values (minimum r). Tables 3-36 and 3-50 of part 6 of the LRFD. Also we can use the column tables in part 2 of the LRFD where we calculate K_yL_y and find P_u from the tables.

Maximum slenderness ratio: for compression members KL/r < 200

Note:

$F_e = \Pi\text{^}2E/(KL/r)\text{^}2$ gives the least F_e to cause buckling. Therefore, KL/r should be maximum and hence r should be minimum. In most cases r_y is the smallest and $(KL/r)_y$ is used. However if long columns are used and bracing is made laterally (perpendicular to weak axis) both $(KL/r)_x$ and $(KL/r)_y$ should be calculated and the largest number is to be used in calculating F_e.

CHAPTER 6

DESIGN OF AXIALLY LOADED COMPRESSION MEMBERS

<u>Introduction:</u> two methods are used in designing axially loaded compression members:

Method 1:

A trial and error process is used to design columns, by using formulas, because we have to know a column size to find $\phi_c F_{cr}$.
We can assume a design stress ($\sigma = F/A$), by assuming a KL/r value and going to the tables. Then divide the factored load by that assumed stress to find the area required. Use that area calculated to select a column and check its ability to carry the load.

Assuming a KL/r value requires a lot of experience.

- ***For columns between 10 to 15 feet KL/r = 40 to 60**
- ***For longer columns KL/r is a little higher.**
- ***For larger P_u (750 to 1000k) we need a large column with larger r and hence smaller KL/r.**
- ***For lightly loaded bracing members KL/r can be over a 100.**

<u>PROCEDURE:</u>

1) **Find P_u**
2) **Assume KL/r**
3) **Find $\phi_c F_{cr}$ using tables 3-36 and 3-50**
4) **Find required area = $P_u/\phi_c F_{cr}$**
5) **Find a section**
6) **Find KL/r_{min} of chosen section**
7) **Find $\phi_c F_{cr}$ from tables 3-36 and 3-50**
8) **Find $\phi_c P_n = \phi_c F_{cr} \times$ Area**
9) **If $\phi_c P_n \geq P_u$ then the section is good**
10) **If $\phi_c P_n \leq P_u$ then try another section**

Method 2:

Using the tables in section 3 of the LRFD manual eliminates the need for a trial and error process. The tables directly gives the value of $\phi_c P_n$ for different steel sections based on the least radius of gyration (which is r_y in most cases and hence making KL/r_y control the design). If KL/r_x controls then a different approach has to be used for design.

PROCEDURE:

1) Choose the KL value in the weaker direction.
2) Knowing $P_u = \phi_c P_n$, enter the tables from the left with KL and move to the right until a $\phi_c P_n$ value is found close to what we have.
3) The section above that value of $\phi_c P_n$ is the desired section.

For braced columns: 1) We can use trial and error like before, however, the largest of $(KL/r)_x$ and $(KL/r)_y$ is to be used. Therefore, to simplify the problem we can assume that K and the strength are the same in both directions, then if $L_x/r_x = L_y/r_y$ (all unbraced lengths are equal) and $L_x = L_y(r_x/r_y)$ so that L_y is equivalent to L_x

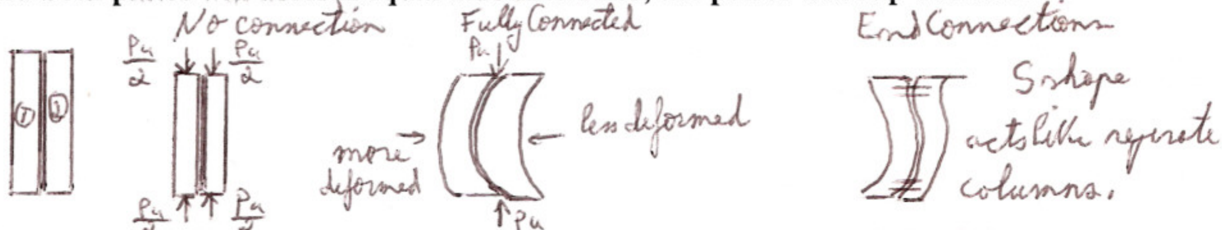
If $L_y(r_x/r_y) < L_x$ then L_x controls

$L_y(r_x/r_y) > L_x$ then L_y controls

If the unbraced lengths are different:

1) Enter tables with K_yL_y and select a section
2) Find r_x/r_y of that shape
3) Multiply r_x/r_y by K_yL_y
4) If $(r_x/r_y)K_yL_y > K_xL_x$ then K_yL_y controls and selected section is good.
5) If $(r_x/r_y)K_yL_y < K_xL_x$ then K_xL_x controls, then reenter tables with $K_xL_x/(r_x/r_y)$ and find the final section.

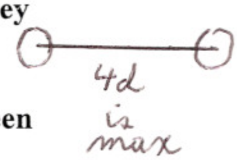
Built-Up Columns With Components in Contact With Each Other: If 2 plates are placed side by side without any connections, then they will act separately and each plate will carry half of the load, and the moment of inertia will be twice that of one plate, and both plates will deform equal amounts. Also, the plates will slip over each other.



If the 2 plates are connected enough to prevent slippage, fully connected along its length, then the moment of inertia is that of the whole cross-section and is four times bigger. Also, in this case the plates deform different amounts. In this case $KL/r = 1.732L$ where $K = 1$ and $r = (I/A)^{0.5}$.

If the plates are connected only at their ends, then $K = 0.5$ and $KL/r = 1.732L$
The last 2 cases have equal design stresses and can carry the same loads as long as the connected one does not start separating.

Connection Requirements for built-up columns whose components are in contact with each other and are bearing on base plates or milled surfaces, Basically the connection is required to transfer stresses. For columns made of more than one section, they have to be connected at their ends, for example the weld length should be greater than or equal to the maximum width of the member, and for bolts they can not be separated by more than 4 diameters on center, and their connection length should be greater than or equal to 1.5 times the maximum width of the member. The LRFD manual also specifies bolted and welded connections between the end connections. Few examples of LRFD specifications are:



1) If an outside plate is part of a built-up section, If we use welds or bolts along gage lines, the maximum spacing should be less than or equal to $[127/(F_y)^{0.5}]$ ×thickness of thinner outside plate or 12 inches
2) If fasteners are staggered on each gage line, then the spacing should be less than or equal to

   $[190/(F_y)^{0.5}]$×thickness of thinner plate or 18 inches.
3) If compression members consists of 2 or more shapes they should be connected together at intervals such that: Ka/r_i (effective slenderness ratio) of each component shape between the connectors is not larger than 0.75 times the controlling slenderness ratio of the whole built-up member. $(KL/r) \times 0.75$

   Where r_i = individual least radius of gyration of an individual component of a column,

   a = distance between connectors.

- Design strength of built-up sections is found by using the LRFD equations on page 2-22 with one exception: If the column does not act as a unit and we have relative deformations in its different parts causing shear stresses at the connections. In this case we will have to modify KL/r of that axis of buckling (section E-4 of the LRFD).

  Equation E4-1 accounts for shear deformations:

  a) For intermediate connectors with snug tight bolts:

     $(KL/r)_m = [(KL/r)_o^2 + (a/r_i)^2]^{0.5}$

  Equation E4-2 for intermediate connectors that are welded or have fully tensioned bolts as required for slip critical joints (more than snug tight):

  $$(KL/r)_m = \{(KL/r)_o^2 + 0.82[(\alpha)^2/(1+\alpha)^2](a/r_{ib})^2\}^{0.5}$$

where $(KL/r)_o$ is the unmodified slenderness of the whole built-up section acting as a unit.

$(KL/r)_m$ is the modified slenderness of built-up members

a = Distance between connectors.

r_i = Minimum radius of gyration of individual components (in)

r_{ib} = Radius of gyration of individual components relative to its centroidal axis parallel to the members axis of buckling (in)

h = Distance between centroids of individual components perpendicular to the members axis of buckling (in)

α = separation ratio = $h/2r_{ib}$

$(Kl/r)_m$ about axis where buckling can occur has to be found if shear can occur at the connections, and it should be checked if it effects the design strength of the member, if it does effect the design strength then the member sizes may have to be changed.

<u>Built-Up Columns With Components Not in Contact With Each Other:</u> Since components of members are not in contact, they have to be connected to each other.

The connection can be made of: Refer to page 177 figure 6-9

a) Continuous cover plates with perforated holes for access purposes. LRFD E-4 for specifications.

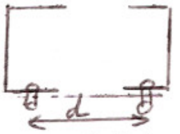
b) End and intermediate tie plates plus lacings.

b) Battens not covered by LRFD.

In all types of connections, the connections should keep the members parallel to each other, prevent individual component buckling, keep distances between components fixed, and make built up section act as a unity.

Note:

- While lacing, the L/r of components between connections can not exceed the governing KL/r of the entire cross-section.
- Lacing is subjected to a shear force normal to the member and equal to not less than 2% of $\phi_c P_n$ of the member.
- Single lacing has slenderness ratio limited to 140
- Double lacing has slenderness ratio limited to 200
- Lacings are designed using column formulas
- If the distance between connection lines is greater than 15 in, then use double lacing. Otherwise use single lacing.



<u>Procedure:</u>

1) Find distance between connections and determine if single or double lacing is needed.
2) From the geometry of the figure assume the angle of lace and determine its length (length of lace or brace)
3) Find L/r of lace and check if it is less than L/r of the entire cross-section
4) Find load on lace which is $V_u = 0.02\ \phi_c P_n$ of the member
5) Find shear force on each plane of lacing $(V_u/2)$
6) Find the force in the bar $[\cos\alpha(V_u/2)]$

7) Find properties of lace bar (moment of inertia, area, radius of gyration) in terms of the thickness and width.

8) Design the bar: assume L/r = x and plug the values found in step 7 into L/r, the only unknown is the thickness t, so calculate t.
9) Check by finding L/r using the t calculated in step 8, find $\phi_c F_{cr}$ from tables, find required area = (Force in bars from step 6)/ $\phi_c F_{cr}$, find required dimensions and end spacings.

Buckling can occur in three different ways:

1) Flexural buckling or Euler's buckling , this is the buckling we talked about so far.
2) Torsional buckling is very complex and is avoided by careful arrangements of members, and by bracing to prevent lateral movement and twisting, and by providing sufficient end supports. (Tables for W, M, S, tube and pipe sections are based on flexural buckling. If torsional buckling is to be accounted for, boxed sections work best; also shorter members will be more efficient.
3) Flexural Torsional buckling For singly symmetrical sections such as Tee or Double angle. This type of buckling can occur and control the design. For unequal leg single angle column, flexural-torsional buckling always controls. Column tables for these sections are due to buckling about the weaker of x, y axis and for flexural torsional buckling (Appendix D in book has an example of such buckling).

CHAPTER 7

DESIGN OF AXIALLY LOADED COMPRESSION MEMBERS CONTINUED

Further Discussion of Effective length: The K values discussed in chapter 5, may not be similar to real life conditions (end restraint conditions), and are used for preliminary designs and for columns that are braced against sidesway. This chapter cover K values for columns in frames, columns subjected to sidesway, and columns that are braced.

Sidesway related to K values: Sidesway is a type of buckling, it occurs where frames deflect laterally due to the presence of lateral loads or unsymmetrical vertical loads or at ends of columns that can move transversely when loaded until buckling occur. Sidesway can be prevented by bracing or by shear walls between columns.

The LRFD specifications C2 states that $K = 1$ is to be used for columns in frames with sidesway unless a proof can be shown for using smaller values.

The following discussion is the proof: this principle is based on the fact that K values of a column depend on the whole structure, which the column is a part of.

Referring to figure 7-2 page 189: Part a) of the figure gives K values where sidesway is prevented by bracing's or shear walls.

Part b) of the figure give K values where sidesway is uninhibited, that is stiffness comes from the structure only.

To use those charts in finding K values, preliminary column and girder sizes are needed.

Procedure for finding K values using the charts of figure 7-2:

1) Locate the 2 joints of the column.
2) For every joint calculate: $G_A = \Sigma(I_c/L_c)/\Sigma(I_g/L_g)$ and $G_B = \Sigma(I_c/L_c)/\Sigma(I_g/L_g)$

The letter "c" stands for columns, whereas the letter "g" stands for girders.
Therefore we add up (I_c/L_c) for every column attached to that joint and divide that result by the summation of (I_g/L_g) of every girder attached to that same joint.

3) Plug the values of G_A and G_B in the charts of figure 7-2 and join them by a straight line.
4) Read the K value at the intersection point between the straight line of step 3 and the middle column of the chart.

Those alignment charts are developed from a slope deflection analysis and the fact that the resistance to rotation provided by the beams and girders at one end of a column depends on the rotational stiffness of those members.

Recommendation for using the alignment charts: For end supports

1) Theoretically $G = \infty$ for pin connected columns. However, a value of 10 is used because friction occurs.
2) Theoretically $G = 0$ for rigid column supports. However, a value of 1 is used since there is no perfectly fixed ends.
3) For beams and girders rigidly attached to columns multiply their I/L by the factors of table 7-1 page 192.

Stiffness Reduction Factors: the K values from alignment charts are based on many assumptions (elastic column behavior, all columns buckle simultaneously, members have constant cross-sections, all joints are rigid…), such assumptions make the K values very conservative and hence needs to be corrected.

In the elastic stage, column stiffness is proportional to EI where E is the modulus of elasticity (29,000 Ksi). In the inelastic stage, column stiffness is proportional to E_rI where E_r is the reduced or tangent modulus (smaller than E). Therefore, for inelastic behavior, E_r is used reducing G and hence reducing K.

Therefore, if the alignment charts have to be used for inelastic behavior,

The G value has to be multiplied by a reduction factor SRF where:

$$(SRF) = E_r/E \cong F_{cr}\text{ inelastic}/ F_{cr}\text{ elastic} \cong (P_u/A)/ F_{cr}\text{ elastic}$$

SRF values for different P_u/A values are in table 3-1 of the LRFD manual. If P_u/A is smaller than the tables, it is in the elastic range.

For inelastic buckling the procedure is as follows:

1) find P_u and a trial column size.
2) Find P_u/A and find SRF
3) Find G elastic for the column and multiply them by SRF and use those values in the alignment charts to find K.
4) Find KL/r and $\phi_c F_{cr}$
5) Find $P_u = \phi_c F_{cr} \times A$
6) Check P_u of step 5, if it is greater than that of step 1 then column is good.
7) If P_u of step 5 is less than that of step 1, then try a different column size.

Columns Leaning on Each Other for In Plane Design: For unbraced frames with beams rigidly attached to the columns, we design each column individually using sidesway uninhibited alignment charts. Those charts are based on the assumption that if one column gets ready to buckle, all the columns at that level will also be ready to buckle and hence they can not support each other. However, sometimes due to different loading conditions the exterior columns have more buckling strength than the interior columns, and when the interior columns start to buckle the exterior ones will hold them.

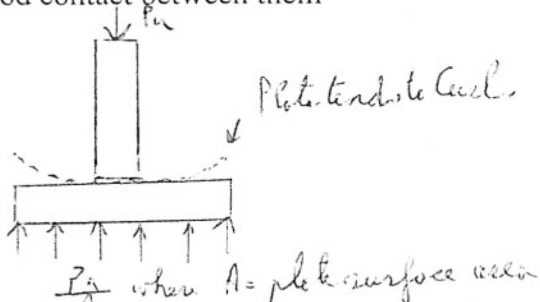
A pin-ended column does not support lateral stability and is called leaning column (it depends on other parts of column).

Base Plates For Concentrically Loaded Columns: The base plates job is to transfer the compressive load on the columns to a large area of the footing because the footings (concrete or soil) compressive strength is usually smaller than that of the column (figure 7-11 page 202 gives types of connections).

Column base plates have to be properly positioned and leveled in place. This is done as follows:

a) For base plates of 20 to 22 inches we use leveling plates (0.25 inches thick and same size as the plate) that are set at right elevation where the column will be placed with its base plate on top of it.
b) For 36 inch base plates, leveling nuts on four anchor bolts are used to adjust the plate up or down.
c) For plates greater than 36 inches, the plates are placed in advance and leveled with shims and wedges, then the column is based on top of it (done that way because they are too heavy).

To ensure complete load transfer from column to plate, Good contact between them should occur, Section M2.8 of the LRFD.



Approximately: Maximum moments in a base plate occurs at distances equal to $0.8b_f$ and $0.95d$ apart (figure 7-12). Find the bending moment at each of these sections and the largest moment controls in finding the area of the base plate.

Plate Area: a) If plate covers entire area of concrete beneath it, the design strength of concrete should at least be:

$$P_u = \phi_c P_p = \phi_c(0.85f'_c)(A_1)$$

Where A_1 = Area of plate

f'_c = Compressive strength of concrete at 28 days

$\phi_c = 0.6$

Therefore, $A_1 = P_u/\phi_c 0.85f'_c$

b) If area of concrete is greater than area of base plate(concrete under plate is stronger because concrete outside the plate prevents it from movement). Therefore, design strength of concrete is higher and this is accounted for by multiplying $\phi_c(0.85f'_c)(A_1)$ by $(A_2/A_1)^{0.5}$ that should be less than or equal to 2.
A_2 = total area of concrete.

Therefore $\phi_c P_p = \phi_c(0.85f'_c)(A_1)(A_2/A_1)^{0.5}$ where $(A_2/A_1)^{0.5} \leq 2$

Therefore $A_1 = P_u/\phi_c(0.85f'_c)(A_2/A_1)^{0.5}$ where $(A_2/A_1)^{0.5}$ may not be greater than 2

Note: A_1 should be $\geq$ column depth $\times$ flange width = column dimensions.

Once A_1, the controlling value, is found, the plate dimensions have to be optimized. To do that the thickness of the plate is kept at a minimum if m = n of figure 7-12 and this occurs if the following equation is satisfied: $B \cong A_1/N$ where $N = (A_1)^{0.5} + \Delta$
A_1 = area of plate = BN and $\Delta = 0.5(0.95d - 0.8b_f)$

Plate Thickness: W.A. Thortons method.
The plate thickness is determined using $l = \max(m, n, \text{or } \lambda_{n'})$

Where $t = l(2P_u/0.9F_yBN)^{0.5}$

And $\lambda_n' = \lambda(db_f)^{0.5}/4$ where $\lambda = 2(X)^{0.5}/1 + (1-X)^{0.5} \leq 1$

Where $X = [4db_f/(d + b_f)^2] \times P_u/\phi_cP_p$

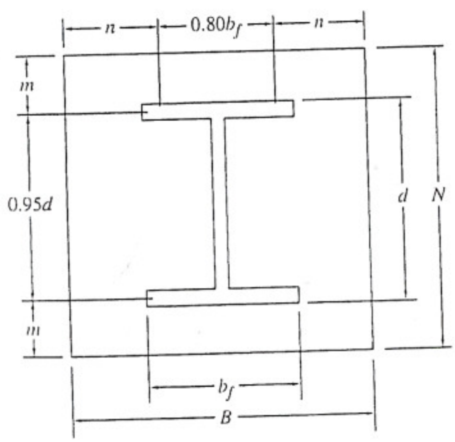
If plate covers all the concrete then $\phi_cP_p = \phi_c 0.85f'_cA_1$
If concrete area is greater than area of plate then $\phi_pP_p = \phi_c f'_c A_1(A_2/A_1)^{0.5}$
where $(A_2/A_1)^{0.5} \leq 2$

General Procedure for Analysis:

1) Find A_1 using the formula that applies to the given situation.
2) Make sure it is at least as large as the column (db_f)
3) Optimize base plate dimensions using $N = (A_1)^{0.5} + \Delta$ and $B = A_1/N$
4) Find thickness (figure 7-12) $m = (N - 0.95d)/2$
   Where $n = (B - 0.85b_f)/2$

And find λ_n' as described before
Take the maximum value of step 4 and find

$$t = l(2P_u/0.9F_yBN)^{0.5}$$



CHAPTER 8

INTRODUCTION TO BEAMS

Types of Beams: Beams usually are horizontal and they support vertical loads. Different types of beams are: a) Joists that support floor and roof loads.

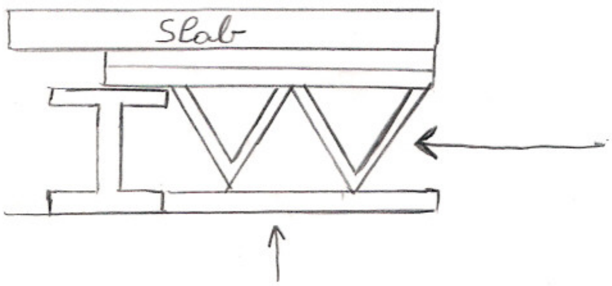
b) Lintels that are beams over openings in masonry walls (windows and doors).

c) Spandrels that support exterior building walls.

d) Stringers are used in road construction and are parallel to the road while floor beams are perpendicular to the road.

Sections Used as Beams: W sections are the most economical and most widely used. W shapes have more steel in their flanges than S sections do and that increases their moment of inertia and hence increases their load carrying capacity.

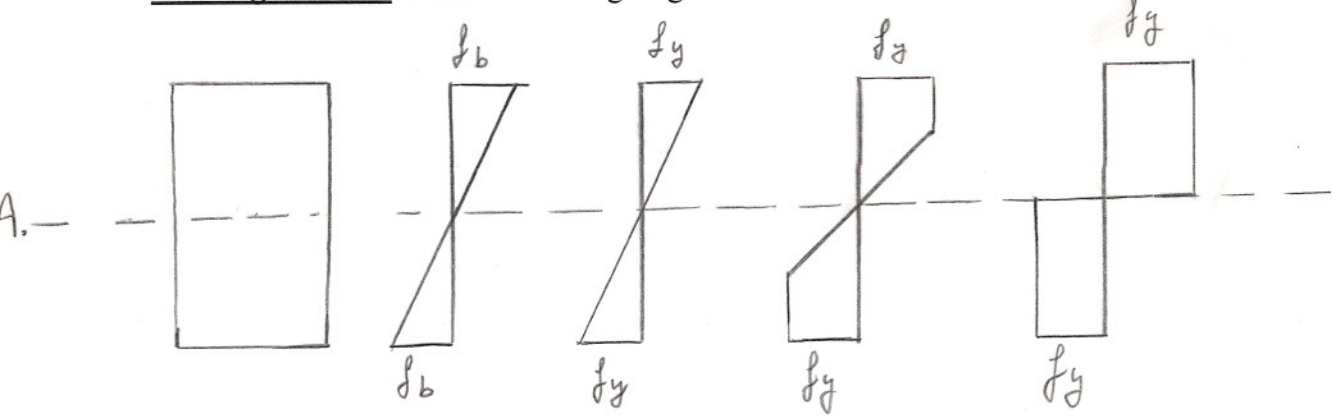
Another common type of beams is open web joists or bar joists used for supporting lightweight slabs.



Made of steel bars, channels or any other rolled shape

Bottom roof

Bending Stresses: If lateral buckling is ignored.



Stress calculated from flexural formulas gives the stress $f_b < f_y$ (beam is below elastic limit)

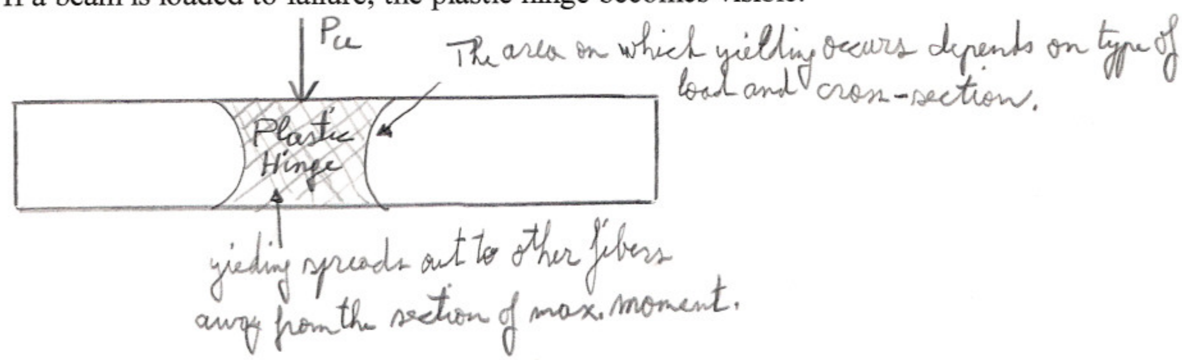
$f_b = Mc/I$ where I/c = Section Modulus = S = Constant

Therefore $f_b = M/S$

Until f_y is reached, the stress increases linearly .
The yield moment is the moment that produces f_y in the outermost fiber of the section. As the moment increase past the yield moment, the outermost fiber yields and the fiber closest to the neutral axis has to resist the load. The higher the moment, the more fibers yields until all the fibers yield and a Plastic Hinge is formed (no extra moment can be resisted) at the plastic moment (last figure in the above diagram).

$M_{plastic}/M_{yielding}$ = Shape Factor\

Plastic Hinges: For analysis purposes, we assume that the plastic hinge is concentrated on one section, however, it actually spreads out to other adjacent fibers as shown on the figure below. If a beam is loaded to failure, the plastic hinge becomes visible.



Elastic Design: Elastic theory designs are based on the load that will first cause a stress somewhere in the structure to equal the yield stress. However, due to extensive research, we know that a great deal of yielding has to occur before failure exists, therefore, the elastic theory is considered to be too safe.

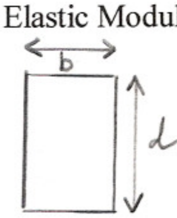
The Plastic Modulus: The yield moment $M_y = f_y \times S$

where S = Elastic Modulus = I/c

For a rectangular section, $I = bd^3/12$ and $c = d/2$

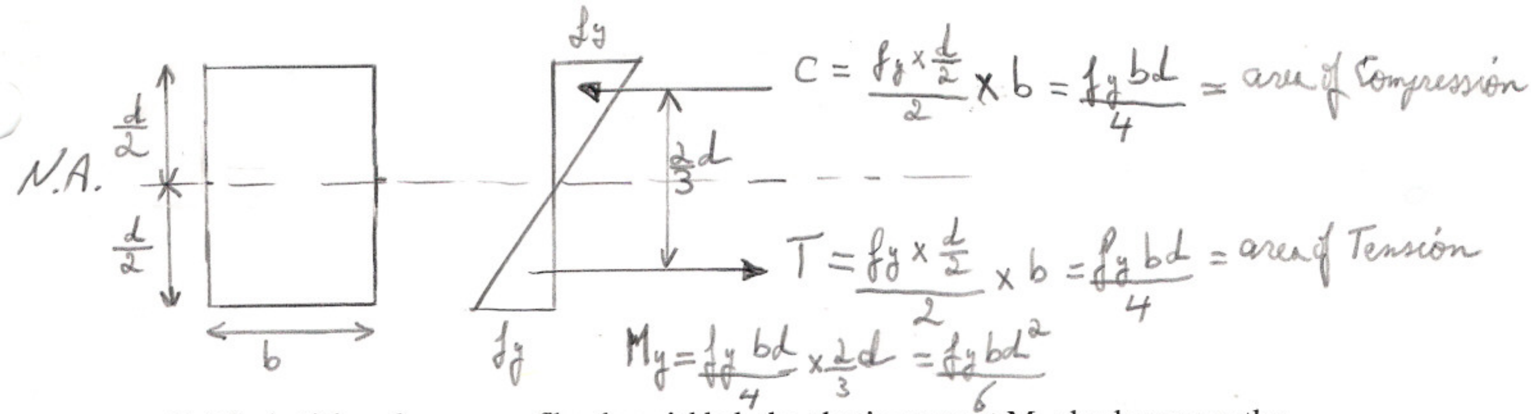
Therefore $S = bd^2/6$

and $M_y = f_y \times bd^2/6$

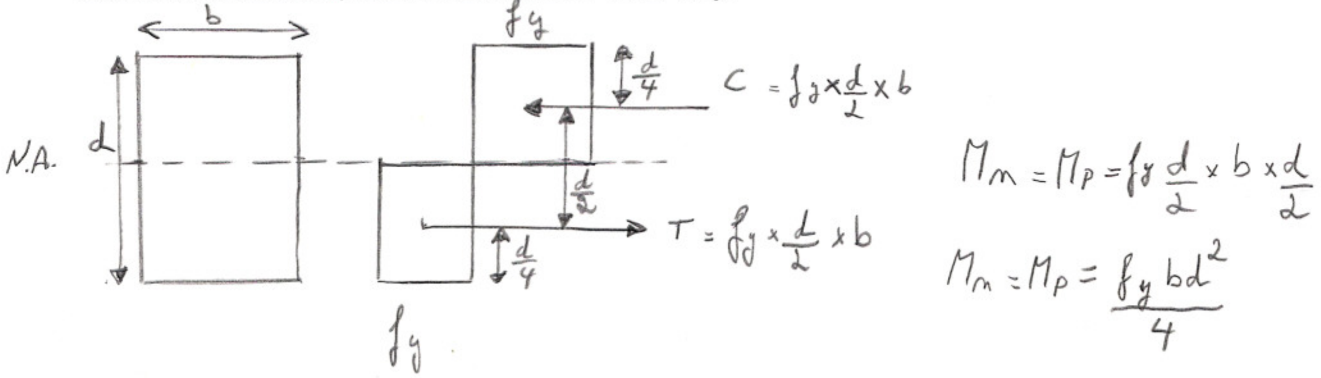


Another method used to find M_y is by calculating the moment created by the couple developed from the tension and compression forces on the section.

M_y = Moment created by the couple



At full plasticity where every fiber has yielded, the plastic moment M_p, also known as the nominal moment M_n, can be found in the same way.

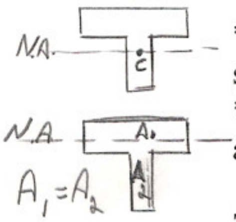


Or we can also find $M_p = f_y \times Z$ where Z is the plastic modulus
For a rectangular section Z = bd^2/4

The shape factor = $M_n/M_y = f_yZ/f_yS = Z/S$

For a rectangular section the Shape factor = (bd^2/4)/(bd^2/6) = 1.5

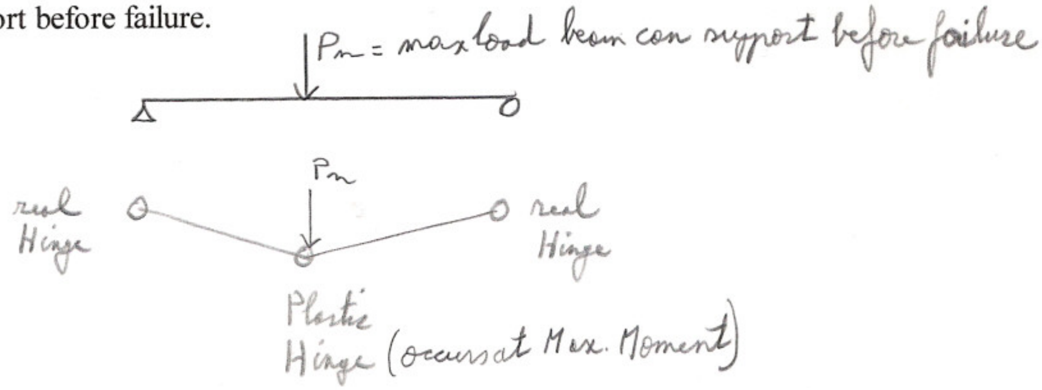
Note: For unsymmetrical sections, the neutral axis of the elastic condition is not the same as that for the plastic condition.



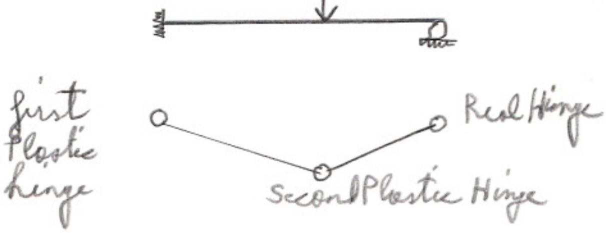
* For the elastic condition, the neutral axis coincides with the neutral axis of the entire section.
* For the plastic condition the neutral axis is located such that the area above the neutral axis equals the area below the neutral axis.

The Collapse Mechanism:

1) A statically determinate beam will fail if **One** plastic hinge develops. The plastic hinge occurs at the location of the maximum moment developed by the maximum load a beam can support before failure.



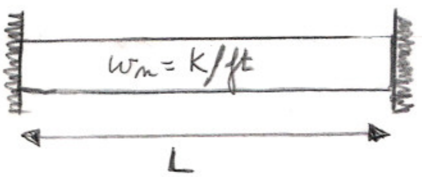
2) A statically indeterminate structure requires at least 2 plastic hinges to fail. This number of plastic hinges varies from structure to structure.



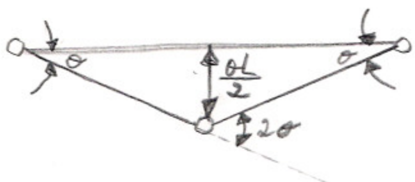
Virtual Work Method: used for plastic analysis of structures.

1) Load the structure to its nominal capacity M_n.
2) As M_n is reached, we assume that a small additional displacement occurs.
3) The work performed by the external loads during this displacement is equated to the internal work absorbed by the hinges.

Example:



3 plastic hinges will develop.



From geometry and symmetry determine the rotation angles at each hinge. $\theta_{rad} = \tan\theta = \sin\theta$

The work performed by external distributed loads = load × average deflection of mechanism

The work performed by external concentrated loads = load × actual deflection under load.

For our example work = $w_nL \times (\theta L/2)/2 = w_nL \times \theta L/4$

Also, The internal work absorbed at the plastic hinges

=

ΣM_n at each plastic hinge × angle of rotation at that hinge

Foe our example, internal work absorbed at the plastic hinges = $M_n(\theta + 2\theta + \theta)$

Next we equate the work performed to the internal work absorbed by the plastic hinges.

$$w_nL \times \theta L/4 = M_n(\theta + 2\theta + \theta)$$

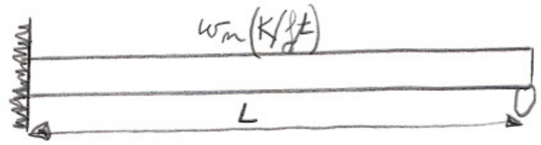
Therefore, $M_n = w_nL^2/16$

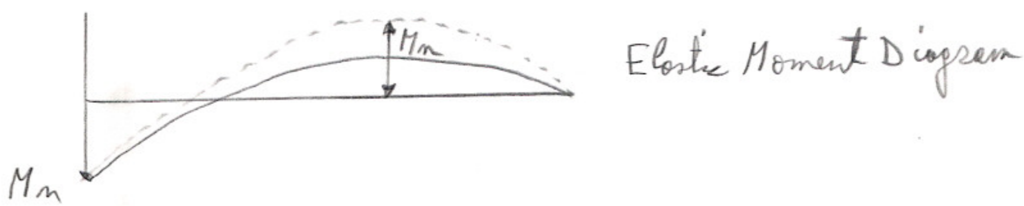
or $w_n = 16M_n/L^2$

Note: A system may require the drawing of several mechanisms to find which one will control. The case that controls will have: P_n is smallest in terms of M_n

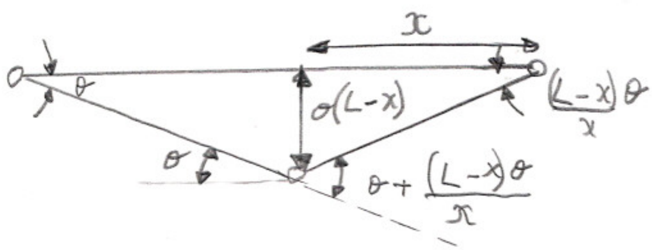
or M_n is greatest in terms of P_n

Location of Plastic Hinges for Uniform Loading: when the beam is simply supported or when the beam is continuous.





As the load increases, plastic hinge first forms at the fixed end, and then the moment will change until another plastic hinge forms at a distance x from the right side.



Using the virtual work method (only include plastic hinges):

$$M_n[\theta + (\theta + (\{L-x\}/x)\theta)] = w_nL[\theta(L - x)]0.5$$

Next we solve for M_n, then find the first time derivative of M_n in terms of x (dM_n/dx), this value equals to the shear. However, at maximum moment the shear equals to zero.

Therefore by setting $dM_n/dx = 0$ will be used to calculate x.

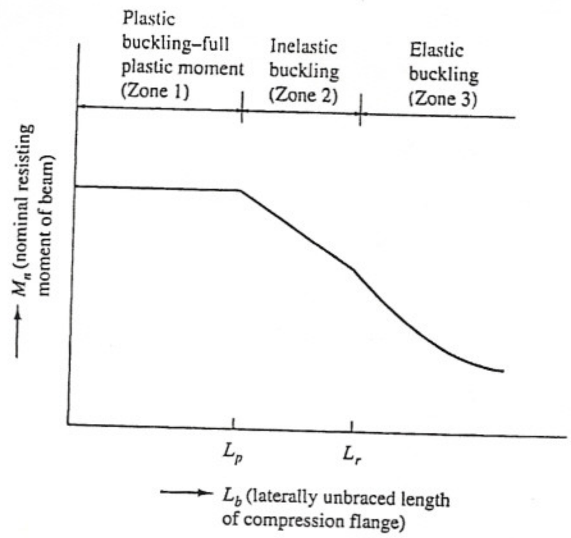
For our example $x = 0.414L$

Continuous Beams: are solved using the same approach as that used for single spans, however, each span is considered separately. Refer to examples.

CHAPTER 9

DESIGN OF BEAMS FOR MOMENTS

Introduction: When a beam is loaded, the section above the neutral axis will be in compression, and hence acts as a column such that as the bracing length increases, the resisting beam moment decreases. We have 3 zones of buckling.



a) Zone 1: Plastic Buckling: When we have continuous lateral bracing of the compression flange. Most beams fall in this zone.
If the spacing or the lateral unbraced length $\leq L_p$ then we are in Zone 1 (L_p depends on beams cross-section and F_y)

Therefore, the beam can be loaded until M_p (plastic moment) is reached and it also can be loaded past that where moments redistribution occurs because the lateral bracing prevents the rotation of the beam allowing it to carry more load. (Chapter 8 where all fibers reaches F_y)

b) Zone 2: Inelastic Buckling: bracing at short intervals: We can load the member until some and not all its fibers reaches F_y (no full moment redistribution).
This inelastic range ends when F_y is reached at only one point or location on the beam. This controls the maximum unbraced length we can have and still be in the inelastic range. This length equals L_r (L_r depends on the cross-section, F_y and residual stresses).
Therefore, at an unbraced length L_r, as soon as an F_y is created at any location, buckling occurs. However, usually buckling occurs before F_y is reached due to residual stresses.

c) Zone 3: Elastic Buckling: braced at large intervals. $L_b > L_r$

In this case buckling occurs before F_y is reached anywhere. As the moment increases, M_{cr} is the moment that causes the section to twist and the compression flange to move laterally (discussed later in the chapter).

Plastic Buckling: Zone 1: for compact I or C shaped sections.

If $L_b \leq L_p$ (for elastic analysis)

where L_b = unbraced length
And $L_p = 300r_y/(F_{yf})$^0.5

And if
$L_b \leq L_{pd}$ (if plastic analysis is used)

where F_{yf} is the specified minimum yield stress in the flange, ksi

where $L_{pd} = [3600+ 2200(M_1/M_2)]r_y/F_y$

Then

$$M_n = M_p = F_yZ \leq 1.5M_y$$
And
$$M_u = \phi_bM_n = 0.9\ F_yZ$$

In the above equations, M_1 is the smallest moment at the end of the unbraced length, and M_2 is the largest moment. M_1/M_2 is positive for double curvature bending 
And it is negative for single curvature bending. 
Also, the equations apply for $F_y \leq 65$ ksi.
The value of M_p is limited to a maximum of $1.5M_y$ to limit the large deformations that can occur when the shape factor is larger than 1.5.
Note: The plastic moment in Zone 1 is not effected by residual stresses because the compressive residual stresses equals the tensile residual stresses and hence cancel their effects.

Design of Beams: Zone 1: To design beams some of the considerations should be: 1) moments 2) Shears
3) Deflections 4) Lateral bracing of compression flanges 5) Fatigue

Step 1 in design is to select a beam that has enough moment capacity to carry (ϕ_bM_n), the factored moment that needs to be carried.
Step 2 is to check all the other criteria to see if they have any effect. The manual starting at page 4-15 gives different sections to be used for different Z_x plastic moduli.

To select a shape we should: 1) Select the lightest member since it is bought by the lb.
2) Z_x on page 4-15 of the manual are for the horizontal axis for beams in their upright position. If the section is turned on its side, its Z is found in the property tables.

Beam weight estimates: A student can not estimate a weight by just guessing.

1) Calculate the maximum bending moment without the beam weight.
2) Use that moment In the tables and pick a section.
3) Use the weight of that section or a little more as your estimated beam weight.

Note: If $L_b = 0$ then we have full lateral support for compression flange. Then we will:

1) Assume or estimate beam weight
2) Find M_u according to type of loading
3) Find $Z_x = M_u/0.9F_y$ and go to tables in section 4 of the LRFD manual and find the cross-section required.

Holes in Beams: need to be avoided if possible. If it is not possible to avoid them then:

1) Place the holes in the web if shear is small (holes in web reduces shear strength)
2) Place the holes in the flange if the moment is small

Theoretically bolt holes will shift the neutral axis from its position. However, tests do not show any considerable effects to be considered, and failure in steel beams depends on the strength of the compression flange.
Flexural test shows that although holes are present in the tension flange, failure occurs in the compression flange. Also, bolt holes in the web will not reduce the value of Z enough to make a difference.
Beam strength with no holes and that of beams with holes in them that do not exceed 15% of the gross area of either flange is almost the same. Therefore, strength of no holes equals strength of 15% holes.
LRFD specifications: Do not subtract area of holes if it is 15% or less than the area of either flange. If the area is > 15% only subtract what exceeds the 15%.
Most practices are more conservative and do subtract the area, even if holes are in one flange they subtract holes like if they are in both flanges.

<u>Lateral Support of Beams:</u> Most beams have their compression flanges restraint laterally against buckling. For example it can be restrained by the concrete slab that it supports on top of it. That makes the beam fall into Zone 1. If it is not laterally supported it reacts like a column in terms of buckling, and this buckling can be effected by residual stresses, spacing of lateral supports, type of supports or restraints, type of materials, and type of loading. The tensile flange prevents buckling until the bending moment increases enough to let the compression flange buckle. Once buckling starts torsion occurs, and the smaller the torsional strength of the section (W and S shapes do not have high torsional strength) the faster it fails.
Built-up boxed shapes resist torsion perfectly.

<u>Inelastic buckling: Zone 2:</u>

1) Compression flange is supported laterally at intervals.

2) Member is bent until the yield strain is reached in some but not all of its compression elements before lateral buckling occur.
3) Bracing is not enough to permit the beam to reach full plastic strain distribution before buckling occurs.

4)Due to residual stresses yielding will begin in a section at applied stresses equal to: $\mathbf{F_{yw} - F_r}$ where F_{yw} is the web yield stress and F_r is the compressive residual stress = 10 ksi for rolled shapes and 16.5 ksi for welded shapes

5)If L_b (for I or C sections) > L_p and less than Lr then we will have inelastic failure

If $L_b > L_r$ then we will have elastic failure before F_y is reached (Zone 3)

6)Bending coefficients used in formulas:

C_b is used to account for the effect of end restraints and types of loading on the lateral buckling.

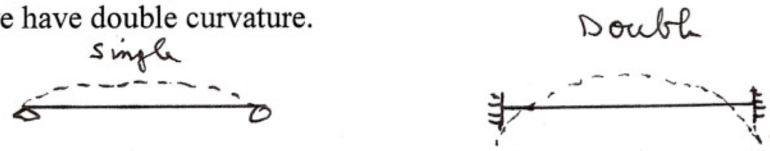
$$C_b = \text{moment coefficient} = 1.75 + 1.05(M_1/M_2) + 0.3(M_1/M_2)^2 \leq 2.3$$

$M_2 > M_1$ and they are the bending moments at the ends of the unbraced length about the strong axis of the member.

If the moment within the unbraced length is > M_1 or M_2 then $C_b = 1$

Part 6 of the LRFD manual gives values of C_b for different M_1/M_2 values.

Note: The equations of Zone 2 and Zone 3 where developed based on $C_b = 1$. However, the LRFD provides $C_b > 1$ to multiply M_n with, hence increasing the moment capacity because $C_b = 1$ is for single curvature and in most cases we have double curvature.



L in compression is shorter for double curvature than it is for single curvature.

Hence, by increasing the moment capacity we reduce cost. However, C_bM_n may not be > than $M_p = F_yZ$ of Zone 1

Equation F1-3 page 258 can be used to calculate C_b

<u>Moment Capacity of Zone 2:</u> as the unbraced length of the compression flange increases beyond L_p, the moment capacity decreases and decreases until L_r (unbraced length) is reached and elastic buckling occurs as F_y is reached.

For I or C shaped sections: If $L_b = L_r$ then $M_u = \phi_b M_r = \phi_s S_x(F_{yw} - F_r)$

L_r is a function of several properties of the beam such as cross-sections, modulus of elasticity, yield stress, torsional strength…

For the unbraced length between L_p and L_r : and for $C_b = 1$

$$\phi_b M_n = C_b[\phi_b M_p - BF(L_b - L_p)] \leq \phi_b M_p$$

Bf is a factor found in the LRFD for every section.

Procedure: for determining the moment capacity of a beam whose $L_p < L_b < L_r$

1) Find L_p, L_r, $\phi_b M_n$, $\phi_b M_p$, and BF from table 4-15 in the LRFD manual
2) Compare L_b given to L_p and L_r.
   If $L_p < L_b < L_r$ then we have inelastic buckling.
3) $\phi_b M_n = C_b[\phi_b M_p - BF(L_b - L_p)]$

Elastic Buckling: Zone 3: If $L_b > L_r$ then elastic buckling will occur before F_y is reached anywhere. The beam supports the moment by bending about its stronger axis until M_{cr} is reached where the beam buckles laterally about its weaker axis. As it bends laterally, the part in tension tries to keep the beam straight causing the beam to twist and buckle laterally. (figure 9-10).

The flexural torsional buckling occurs at :

$M_{cr} = C_b(\pi/L_b)[EI_yGJ + (\pi E/L_b)^2(I_yC_w)]$^0.5

Where G = Shear modulus of elasticity of steel = 11,200 ksi
J = Torsional constant in (inches)^4 in manual tables part1
C_w = Warping constant in (inches)^6 in manual tables

This equation applies for channels and I shapes whereas other sections have different formulas in F1.4 and F1.5

Procedure: for computing $M_u = \phi_b M_{cr}$ for a certain section with a certain L_b

1) Check if $L_b > L_r$ L_r is found in the load factor design tables
2) From the manual get I_y, J, and C_w
4) Use the appropriate formulas.
5) Or we can use part 3 of the LRFD manual where curves are plotted for $\phi_b M_{cr}$ and $\phi_b M_n$ on Page 4-113.

Those charts do not consider factors such as shear, fatigue, and deflection…

6) Or we can use:

$$M_{cr} = \frac{C_b S_x X_1 \sqrt{2}}{L_b/r_y}\sqrt{1 + \frac{X_1^2 X_2}{2(L_b/r_y)^2}}$$

when $X_1 = \frac{\pi}{S_x}\sqrt{\frac{EGJA}{2}}$ and $X_2 = \frac{4C_w}{I_y}\left(\frac{S_x}{GJ}\right)^2$

X_1 and X_2 are related to Section properties.

Design Of Beams
Miscellaneous Topics

Design Of Continuous Beams:

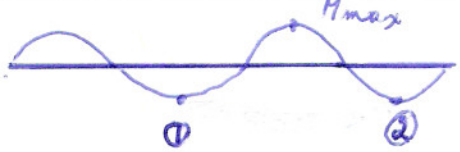
a) Plastic Analysis for sections with $F_y \leq 65$ ksi. This method includes the redistribution of moments caused by overloads if the continuous beam is braced enough laterally on its compression flange.

b) Elastic Analysis that actually estimates the real plastic behavior by taking the design moment equal to the largest of:

(Largest negative moment)×0.9

or

(Largest positive moment) + 0.1×(the average of the two negative moments under the largest positive moment)

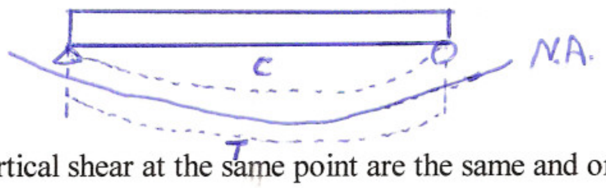


Note: If the moment diagram only has positive moment then the design moment is the maximum positive moment.

Plastic Analysis: Procedure:
a) Do the plastic analysis
b) Find M_u for every span
c) Choose the largest M_u
d) Find $Z_{required} = M_u/0.9F_y$
e) Go to tables and find the section

Elastic Analysis: Procedure:
a) Draw the moment diagram
b) Find M_u using the formulas given above
c) Find $Z_{required} = M_u/0.9F_y$
d) Go to tables and find the section

Shear: As a beam is loaded the different layers of fibers tend to slip of each other.



* Horizontal and vertical shear at the same point are the same and one can not occur without the other.

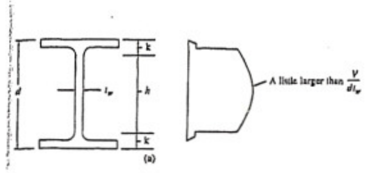
* In steel, usually shear is not a problem because the web of rolled steel can resist large shearing forces.

* Extensive shear forces occur in steel where:

i) Rigidly connected members (columns and beams) whose webs are in the same plane.

ii) Where a beam is notched.

iii) Large concentrated loads are placed at short distances from supports.



Therefore, the shear is basically supported by the web.

As the moment increases, the members will start yielding, and hence part of the web starts yielding reducing its shear carrying capacity. Therefore, we use a reduced shear value and assume it is carried by the entire web area, where area of web = $A_w = d \times t_w$

Shear strength Expressions:

a) Web yielding: includes almost all rolled beam sections.

If $h/t_w \leq 418/(F_{yw})^{0.5}$ where $418/(F_{yw})^{0.5} = 70$ for 36 ksi

$418/(F_{yw})^{0.5} = 59$ for 50 ksi

F_{yw} is the specified minimum yielding stress of the web

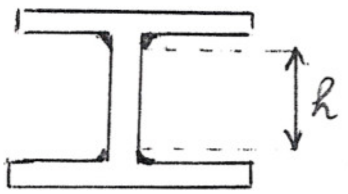
then $V_n = 0.6F_{yw}A_w$

And

$V_u = \phi_u V_n$ where $\phi_u = 0.9$

h = clear distance between fillet.

* For built-up sections that are welded h is the distance between the flanges.
* For built-up sections that are bolted h is the distance between the bolt lines in the web.



b) Inelastic buckling of the web:

If $418/(F_{yw})^{0.5} < h/t_w \leq 523/(F_{yw})^{0.5}$

Where $523/(F_{yw})^{0.5} = 87$ for $F_y = 36$ ksi

$523/(F_{yw})^{0.5} = 74$ for $F_y = 50$ ksi

Then $V_n = 0.6F_{yw}A_w(418/(F_{yw})^{0.5})/(h/t_w)$

And

$V_u = \phi_v V_n$ where $\phi_v = 0.9$

c) Elastic buckling of the web:

If $523/(F_{yw})^{0.5} < h/t_w \leq 260$

Then $V_n = (132{,}000A_w)/(h/t_w)^2$ $V_u = \phi_v V_n$ where $\phi_v = 0.9$

Note: The LRFD manual in section 4 titled "Beam W shapes maximum factored uniform loads in Kips for beams laterally supported" gives values for $\phi_v V_n$ and other information on web yielding.
One way to increase the shear strength of the web, is to bolt plates on both of its sides.

Deflections: in steel beams have to be limited due to:

i) Large deflections can effect other parts of the structure attached to the beam.

ii) Bad appearance and unsafe looks.

Service live load deflections are limited to 1/360 of the span length (that limit prevents cracks in underlying plaster). This value can change with different types of loads, structures and specifications. The deflection limits are decided by the designer's experience.
Deflection can also be limited by limiting the depth span ratios (Table4-2 in LRFD).
Cambering can help reduce deflection but it is expensive.
To determine the deflections:

$\Delta = ML^2/C_1 I_x$

General equation

where M is the moment based on the uniformly distributed service loads
C_1 is a constant figure 10-8
I_x is moment of inertia about x-axis

If deflection controls the design of a beam:

i) Assume a beam weight

ii) Find w_u and M_u

iii) Find $Z_{required}$

iv) Find a section from the tables

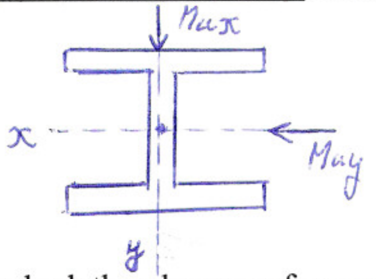
v) Find the actual deflection of the section

vi) If the actual deflection is greater than the required deflection, Calculate the maximum allowed
I_x = (actual deflection/required deflection)(I_x of selected member) that will limit the deflection.

vii) Use the calculated I_x in part 4 of the LRFD "Moment of inertia selection tables" and find a section for the I_x calculated in step vi.

Vibrations: and their control is a very important part of the design.
Damping of vibrations can be controlled by: stiffer structures, partitioning, office furniture...
Ponding: occur when water is not drained as fast as it is pored, this creats deflections that can cause failures. To prevent ponding roofs have to be inclined a minimum of a quarter of an inch per foot, and to prevent failure the roof should be stiff enough.

Unsymmetrical Bending: occurs about an axis other than the principle axis.



If the load is not perpendicular to the principle axis, it is broken into its components about the principle axis and the moment it produces about the principle axis (M_{ux} and M_{uy})

To check the adequacy of a member when it is exposed to bending about both axis and to tension or compression:

If $P_u/\phi P_n < 0.2$

Then

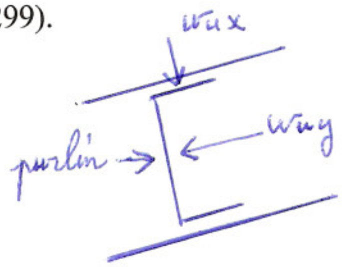
$P_u/2\phi P_n + (M_{ux}/\phi_b M_{nx} + M_{uy}/\phi_b M_{ny}) \leq 1.0$ If it is ⇒ section is good

If $P_u = 0$

Then

$(M_{ux}/\phi_b M_{nx} + M_{uy}/\phi_b M_{ny}) \leq 1.0$ If it is ⇒ Section is good.

Design of Purlins: Purlins are used in roof trusses to avoid bending in the top chord of the roof truss. They are usually spaced 2 to 6 feeet apart depending on weight. Their most desirable depth to span ratio = 1/24. Most common sections are channels or S sections. However, channels and S sections are weak about their web axis and sag rods are sometimes used to reduce the span length for bending about those axis (figure 10-13 page 299).



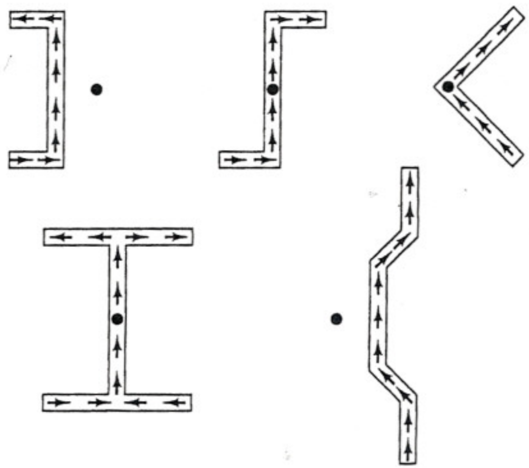
If sag rods are not used, then M_{max} about the web axis $= w_{uy}L^2/8$

where L = distance between trusses

If sag rods are used in middle span then M_{max} about the web axis $= w_{uy}L^2/32$

If sag rods are used at onethird points then M_{max} about the web axis $= w_{uy}L^2/90$

The Shear Center: is defined as the point on the cross-section of a beam through which the resultant of the transverse loads must pass so that the stresses in the beam may be due to only pure bending and transverse shear and not torsional moments. The following figure shows the location of the shear center for few cross-sections. Also shear center locations are provided for channels in the "Properties Table of Part 1 of the LRFD manual".



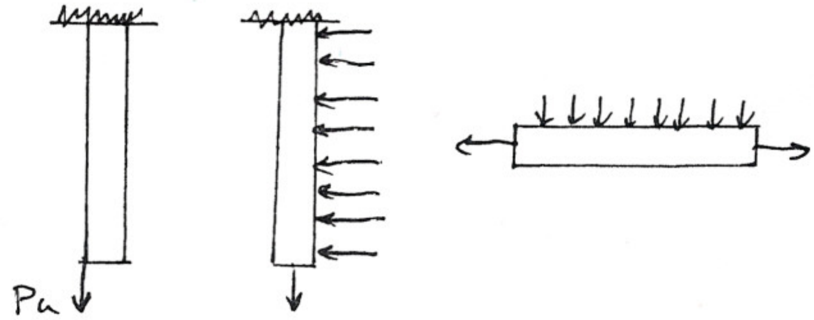
CHAPTER 11

BENDING AND AXIAL FORCE

Bending and axial forces occur together very frequently. For example a column is exposed to both because it is impossible to apply the load exactly at the center of the column and hence we develop bending moments.

Even in trusses bending is taking place, weight of members cause moments. However compression members have more bending because the compressive force tends to bend the member whereas tensile forces tend to reduce lateral deflections. Also winds and traffic create lateral deflections.

Some members exposed to bending and axial forces are:
For tensile axial forces only first-order analysis are needed.



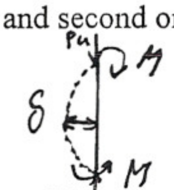
For symmetric shapes subjected to bending and tensile axial forces:

If $P_u/\phi_t P_n \geq 0.2 \Rightarrow P_u/\phi_t P_n + 8/9(M_{ux}/\phi_b M_{nx} + M_{uy}/\phi_b M_{ny}) \leq 1.0$

If $P_u/\phi_t P_n < 0.2 \Rightarrow P_u/2\phi_t P_n + (M_{ux}/\phi_b M_{nx} + M_{uy}/\phi_b M_{ny}) \leq 1.0$

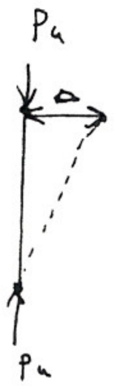
The values in these equations will vary depending on L_b being in which zone.

<u>11-4:</u> First order and second order moments for members subject to axial <u>compression</u> and bending:



The moment M causes a displacement (laterally) of δ. This develops a secondary moment = $P_u\delta$ by which the member moment is increased.
This moment increase causes another lateral deflection, which causes more moment. This goes on until equilibrium is reached.
Finally $M_1 = M_{nt} + P_u\,\delta$
(Assume no lateral translation of frame when finding M_{nt}).

If one end of the column can move:



2^{nd} moment = $P_u \Delta$

and $M_2 = M_{lt} + P_u \Delta$

Where M_{lt} is the moment due to the lateral loads.

To account for the second moments we can:

1) Do the second order analysis.
2) Amplify the moment of the first order elastic analysis.

Therefore, we make two first order analysis.

1) No swaying and find M_{nt}
2) Swaying and find M_{lt}

Then $M_u = B_1 M_{nt} + B_2 M_{lt}$
where B_1 and B_2 are magnification factors.

B_1 estimates $P_u \delta$ for braced column.
B_2 estimates $P_u \Delta$ for unbraced columns.
(This works if connections are fully restrained or fully unrestrained.)

$$B_1 = \frac{C_m}{1 - P_u/P_{e1}} \geq 1.0$$ to magnify and account for $P_u \delta$ and $P_u \Delta$

C_m = modification factor.
P_u = required axial strength of the member.
Where $P_{e1} = \pi$^2EI/(KL)^2 = member Euler bulking strength. I and KL are taken in plane of bending for a braced frame.
B_1 is the magnifier of M_{nt} moments (no lateral translation).

$B_2 = 1/[1- \sum P_u(\Delta_{oh}/ \sum HL)]$ or $B_2 = 1/(1-\sum P_u/ \sum P_{e2})$ if the member size is known.

$\Delta_{oh} = \Delta$ = sway or drift
Δ_{oh}/h = drift index and is limited to provide security for the occupant.
$\sum P_u$ = All required axial strength of all the columns on the same level.
$\sum H$ = Summation of all the story horizontal forces producing Δ_{oh}
$P_{e2} = \pi^2 EI/(KL)^2$

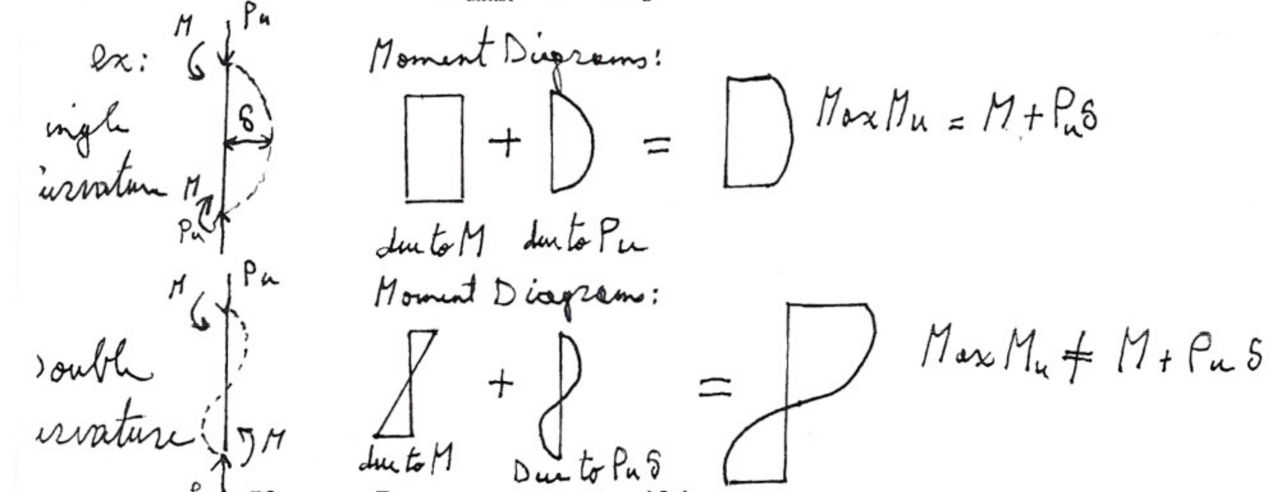
Where, K is determined in plane of bending of unbraced frame

Note : B_2 is for moments that are caused by forces that cause sidesway and is to be computed for an entire story.

<u>11-6</u> Moment Modification or C_m Factors:

B_1 is developed for the largest possible lateral displacement. However a lot of times B_1 over magnifies the columns moments. That is where Cm is used to reduce the moment. We know that if $M_{max} = M_{nt} + P_u\delta \Rightarrow C_m = 1.0$

$$M_{umax} = M \;\; + P_u\delta$$



If we use B1 we are over magnifying
Therefore, we use C_m to account for the over magnification.

<u>Category 1</u>
If we have no joint translation or sidesway and no transverse loading between joint ends :
$C_m = 0.6 - 0.4\, M_1/M_2$ where $M1 < M2$ (moments at ends of unbraced lengths)
M1/M2 is negative for single curvature.
M1/M2 is positive for double curvature.

<u>Category 2 :</u>
Transverse loading between joints does exist.
a) Cm = 0.85 for restrained ends
b) Cm = 1.0 for unrestrained ends.
c) Use rational analysis and interpolate between a) and b).
d) Table 11-1 gives few cases. Review fig.11.7

<u>11-7:</u> Review of Beams-columns in braced frames :

Axial compression and bending or axial tension and bending use the same interaction equation with different meanings for the same terms.
Ex: Pu = tensile force or compressive force.
ϕ_c for compression + 0.85.
ϕ_b for bending = 0.9

Compression and bending require first order analysis(elastic analysis and M_{nt} due to external loads)
And a second order analysis (M_{lt} due to lateral translation)
Theoretically M_{lt}= 0 if(frame and loads are symmetrical and frame is braced.)
=> B_2 is not required.

<u>11-8</u> Review of Beam columns in unbraced frames:
*Maximum primary moment is usually at the end of the columns.
*Total moment = primary moment + sidesway moment
*Modification factor is not used, and Cm is not used in the B_2 expression.

<u>11-9)</u> Design of Beam-column – Braced or Unbraced:

1) Find a trial section.
2) Check it with appropriate interaction equations.
Therefore finding an initial good section can reduce the amount of work to do.
Methods used are :
1) Equivalent axial load or effective axial load method:
a) P_u (axial load) and (M_{ux} and or M_{uy}) bending moment are replaced by their equivalent P_{ueq} which will result in the most economical section just like if M_{ux}, M_y and P_u are being used.

$P_{ueq} = P_u + P_u'$ where P_u' is the equivalent of the bending moments.
$P_{ueq} = P_u + M_{ux}(m) + M_{uy}(mu)$ where 'u' is in the column tables LRFD-3 (see example 3-20 in LRFD): m is a factor in table 11-2 page 339 in book
Once P_{ueq} is found, use concentric column tables in LRFD to design.

<u>Note:</u> First approximation uses the first approximation row in table 11-2 and u = 2. Once the section from that is found, P_{ueq} is solved again using that section's m and u from tables in LRFD. This is repeated until section does not change any more.
b) This P_{ueq} is very conservative and the last step is using the LRFD interaction equations to check the final member.
If we feel it is over designed choose a member one or two sizes smaller and make sure it satisfies the interaction equations.

Note: P_{ueq} method is good if moment is not too large compared to the axial force. If it is, the resulting section will be uneconomical.