6/20

Test Review

(21) Use a cofunction to write an expression equal to $\sin(30°)$.

We know that $\sin(\theta) = \cos\left(\frac{\pi}{2} - \theta\right)$ in radians

or $\sin(\theta) = \cos(90° - \theta)$ in degrees

So $\sin(30°) = \cos(90° - 30°) = \cos(60°)$

(36) Find exact value of $\cos \frac{5\pi}{12}$

$$\frac{5\pi}{12} = \frac{3\pi}{12} + \frac{2\pi}{12}$$

$$\frac{\pi}{4} + \frac{\pi}{6}$$

$$\cos\left(\frac{5\pi}{12}\right) = \cos\left[\frac{\pi}{4} + \frac{\pi}{6}\right] = \cos\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{6}\right) - \sin\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{6}\right)$$

$$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$$

$$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \frac{\sqrt{6} - \sqrt{2}}{4}$$

32) Find exact value of $\tan\left(\frac{19\pi}{12}\right)$

$$\frac{19\pi}{12} = \frac{21\pi}{12} - \frac{2\pi}{12} = \frac{7\pi}{4} - \frac{\pi}{6}$$

$$\tan\left(\frac{7\pi}{4} - \frac{\pi}{6}\right) = \frac{\tan\left(\frac{7\pi}{4}\right) - \tan\left(\frac{\pi}{6}\right)}{1 + \tan\left(\frac{7\pi}{4}\right)\tan\left(\frac{\pi}{6}\right)}$$

$$= \frac{-1 - \frac{\sqrt{3}}{3}}{1 + (-1)\left(\frac{\sqrt{3}}{3}\right)}$$

$$= \frac{-\frac{3}{3} - \frac{\sqrt{3}}{3}}{\frac{3}{3} - \frac{\sqrt{3}}{3}} = \frac{\frac{-3-\sqrt{3}}{3}}{\frac{3-\sqrt{3}}{3}}$$

$$= \frac{-3-\sqrt{3}}{3-\sqrt{3}}$$

Most likely not correct

(34) Find the exact value $\sin(165°)$

$$165 = 120 + 45$$

$$\sin(165°) = \sin(120° + 45°) = \sin(120°)\cos(45°) + \sin(45°)\cos(120°)$$

$$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(-\frac{1}{2}\right)$$

$$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$$

$$= \boxed{\frac{\sqrt{6} - \sqrt{2}}{4}}$$

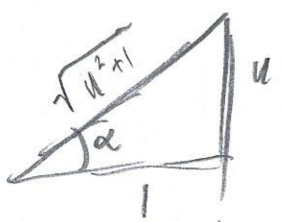
(33) $\sin(\tan^{-1}(u) + \sin^{-1}(v))$

$\alpha = \tan^{-1}(u)$

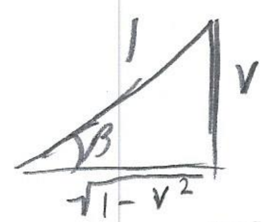
$\beta = \sin^{-1}(v)$

$$\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \sin(\beta)\cos(\alpha)$$

$\tan(\alpha) = u$



$\sin(\beta) = v$



$\cos(\beta) = \sqrt{1-v^2}$

$\sin(\alpha) = \frac{u}{\sqrt{u^2+1}}$

$\cos(\alpha) = \frac{1}{\sqrt{u^2+1}}$

$$\sin(\alpha+\beta) = \left(\frac{u}{\sqrt{u^2+1}}\right)\left(\sqrt{1-v^2}\right) + (v)\left(\frac{1}{\sqrt{u^2+1}}\right)$$

$$= \frac{u\sqrt{1-v^2}}{\sqrt{u^2+1}} + \frac{v}{\sqrt{u^2+1}} = \boxed{\frac{u\sqrt{1-v^2} + v}{\sqrt{u^2+1}}}$$

3

Verify

(32) $\sin\left(x+\frac{\pi}{4}\right)+\sin\left(x-\frac{\pi}{4}\right)=\sqrt{2}\sin x$

$\sin(x)\cos\left(\frac{\pi}{4}\right)+\cos(x)\sin\left(\frac{\pi}{4}\right)+\sin(x)\cos\left(\frac{\pi}{4}\right)-\cos(x)\sin\left(\frac{\pi}{4}\right)=\sqrt{2}\sin x$

$2\sin(x)\cos\left(\frac{\pi}{4}\right)=\sqrt{2}\sin(x)$

$\cancel{2}\sin(x)\left(\frac{\sqrt{2}}{\cancel{2}}\right)=\sqrt{2}\sin(x)$

$\sqrt{2}\sin(x)=\sqrt{2}\sin(x)$

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(31) Verify

$$\frac{1-\cos(-x)}{\sec(-x)-1}=\cos(x)$$

$$\frac{1-\cos(x)}{\sec(x)-1}=\cos(x)$$

$$\frac{1-\cos(x)}{\frac{1}{\cos(x)}-1}=\cos(x)$$

$$\frac{1-\cos(x)}{\frac{1-\cos(x)}{\cos(x)}}=\cos(x)$$

31 cont.

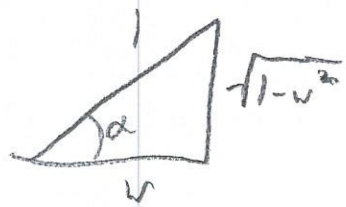
$$\frac{1 \cancel{- \cos(x)}}{1} \cdot \frac{\cos(x)}{\cancel{1-\cos(x)}} = \cos(x)$$

$$\frac{\cos(x)}{1} = \cos(x)$$

29 $\tan(\cos^{-1} w)$

$\alpha = \cos^{-1} w$

$\cos(\alpha) = w$



$$\tan(\alpha) = \frac{\sqrt{1-w^2}}{w}$$

28 Verify

$$\frac{\cos x}{1-\sin x} - \frac{1}{\cos x} = \tan x$$

$$\frac{\cos(x)(1+\sin(x))}{(1-\sin(x))(1+\sin(x))} - \frac{1}{\cos(x)} = \tan(x)$$

$$\frac{\cos(x)(1+\sin(x))}{1-\sin^2(x)} - \frac{1}{\cos(x)} = \tan(x)$$

$$\frac{\cancel{\cos(x)}(1+\sin(x))}{\cos^2(x)} - \frac{1}{\cos(x)} = \tan(x)$$

$$\frac{1+\sin(x)-1}{\cos(x)} = \tan(x)$$

$$\frac{\sin(x)}{\cos(x)} = \tan(x)$$

$$\tan(x) = \tan(x)$$