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3.4 Properties of Logs

Let $b, x,$ and y be positive real numbers where $b \neq 1$ and p be a real number

1) $\log_b 1 = 0$
2) $\log_b b = 1$
3) $\log_b b^p = p$
4) $b^{\log_b x} = x$
5) $\log_b(xy) = \log_b(x) + \log_b(y)$
6) $\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)$
7) $\log_b x^p = p \log_b x$

Change of base Formula

$$\log_N M = \frac{\log_c M}{\log_c N}$$

3.5

Equivalence property of exponential expressions

If b, x, and y are real numbers with $0 < b$ and $b \neq 1$, then

$$b^x = b^y \Rightarrow x = y$$

Ex1: Solve:

$$3^{2x-6} = 81$$

$$3^{2x-6} = 3^4$$

$$2x - 6 = 4$$

$$ +6 \quad +6$$

$$\frac{2x}{2} = \frac{10}{2}$$

$$x = 5$$

step 1: rewrite both sides so that they have the same base.

step 2: set the exponents equal to each other and solve for x.

Ex 2: Solve:

$$25^{-x+3} = 125$$

$$(5^2)^{-x+3} = 5^3$$

$$5^{-2x+6} = 5^3$$

$$-2x + 6 = 3$$

$$ -6 \quad -6$$

$$\frac{-2x}{-2} = \frac{-3}{-2} \Rightarrow x = \frac{3}{2}$$

Ex 3: Solve:

$$16 = 8^{-x+2}$$

$$2^4 = (2^3)^{-x+2}$$

$$2^4 = 2^{-3x+6}$$

$$4 = -3x + 6$$
$$-6 \qquad -6$$

$$\frac{-2}{-3} = \frac{\cancel{-3}x}{\cancel{-3}}$$

$$x = \frac{2}{3}$$

What to do when you cannot make the bases match?

Ex 1: $7^x = 60$ Take the log of both sides

use log rules to solve for x.

$$\log(7^x) = \log(60)$$

$$x\log(7) = \log(60)$$

$$x = \frac{\log(60)}{\log(7)}$$

Ex 2: $17^{10x} = 8^{x-2}$

$$\log(17^{10x}) = \log(8^{x-2})$$

$$(10x)(\log(17)) = (x-2)(\log(8))$$

$$(10x)(\log(17)) = x\log(8) - 2\log(8)$$

$$10x\log(17) - x\log(8) = -2\log(8)$$

$$\frac{x\cancel{(10\log(17) - \log(8))}}{\cancel{(10\log(17) - \log(8))}} = \frac{-2\log(8)}{(10\log(17) - \log(8))}$$

Ex 3: $\log_2(x+3) = -\log_2(x+6) + 2$

$$\log_2(x+3) + \log_2(x+6) = 2$$

$$\log_2((x+3)(x+6)) = 2$$

$$2^2 = (x+3)(x+6)$$

$$4 = x^2 + 9x + 18$$

$$-4 \qquad\qquad -4$$

$$0 = x^2 + 9x + 14$$

$$0 = (x+2)(x+7)$$

$x+7=0$ or $x+2=0$

$x=-7$ or $x=-2$

check: $-7+3 \not> 0$ $\qquad$ $-2+3>0$

$-4 \not> 0$ $\qquad$ $1>0$ ✓

Not a solution $\qquad$ $-2+6>0$

$4>0$ ✓

The solution is 4