3.4 Properties of Logs

Let \( b, x, \) and \( y \) be positive real numbers where \( b \neq 1 \) and \( p \) be a real number.

1) \( \log_b 1 = 0 \)
2) \( \log_b b = 1 \)
3) \( \log_b b^p = p \)
4) \( b^{\log_b x} = x \)

5) \( \log_b (xy) = \log_b (x) + \log_b (y) \)
6) \( \log_b \left( \frac{x}{y} \right) = \log_b (x) - \log_b (y) \)
7) \( \log_b x^p = p \log_b x \)

Change of base formula

\[
\log_b M = \frac{\log_c M}{\log_c N}
\]
3.5

Equivalence property of exponential expressions

If $b, x, and y$ are real numbers with $0 < b$, then

$$b^x = b^y \Rightarrow x = y$$

Ex. 1: Solve:

$$\frac{2x - 6}{3} = 81$$
$$\frac{2x - 6}{3} = 3^4$$
$$2x - 6 = 4\quad +6 +6$$
$$2x = 10 \quad \frac{2x}{2}$$
$$x = 5$$

Step 1: rewrite both sides so that they have the same base.
Step 2: Set the exponents equal to each other and solve for $x$.

Ex 2: Solve:

$$25^{x + 3} = 125$$
$$(5^2)^{x + 3} = 5^3$$
$$5 - 2x + 6 = 5^3$$
$$-2x + 6 = 3\quad -6 -6$$
$$-2x = -3 \quad \frac{-2x}{-2}$$
$$-x = \frac{3}{2} \Rightarrow x = \frac{3}{2}$$
Ex 1: \(7^x = 60\)

- What to do when you cannot make the bases match?
- Take the log of both sides.
- \(\log(7^x) = \log(60)\)
- \(x \log(7) = \log(60)\)
- \(x = \frac{\log(60)}{\log(7)}\)

Ex 2: \(17^{10x} = 8^{x-2}\)

- \(\log(17^{10x}) = \log(8^{x-2})\)
- \((10x) \log(17) = (x-2) \log(8)\)
- \((10x)(\log(17)) = x \log(8) - 2 \log(8)\)
- \(10x \log(17) - x \log(8) = -2 \log(8)\)
- \(x(10 \log(17) - \log(8)) = -2 \log(8)\)
- \(x = \frac{-2 \log(8)}{(10 \log(17) - \log(8))}\)
Ex 3: \[ \log_2 (x+3) = -\log_2 (x+6) + 2 \]

\[ \log_2 (x+3) + \log_2 (x+6) = 2 \]

\[ \log_2 ((x+3)(x+6)) = 2 \]

\[ 2^2 = (x+3)(x+6) \]

\[ 4 = x^2 + 9x + 18 \]

\[ 0 = x^2 + 9x + 14 \]

\[ 0 = (x+2)(x+7) \]

\[ x+7 = 0 \quad \text{or} \quad x+2 = 0 \]

\[ x = -7 \quad \text{or} \quad x = -2 \]

Check:

\[ -7+3 \neq 0 \]

\[ -4 \times 0 \]

\[ \text{Not a solution} \]

\[ -2+3 > 0 \]

\[ 1 > 0 \quad \checkmark \]

\[ -2+6 > 0 \]

\[ 4 > 0 \quad \checkmark \]

The solution is 4.