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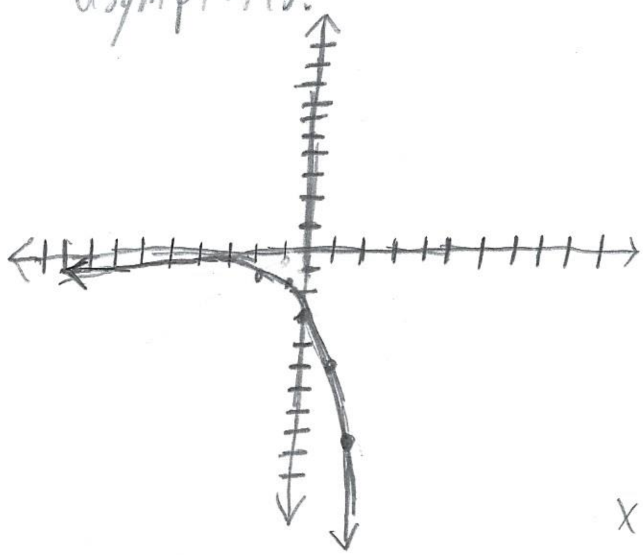
Exponential Functions

$f(x) = -3\left(\frac{5}{3}\right)^x$

graph at the x-values, -2, -1, 0, 1, 2 and draw the asymptotes.



$x = -2$

$-3\left(\frac{5}{3}\right)^{-2}$

$-3\left(\frac{3}{5}\right)^{2}$

$-3\left(\frac{9}{25}\right)$

$y = -\frac{27}{25}$

$x = -1$

$-3\left(\frac{5}{3}\right)^{-1}$

$-3\left(\frac{3}{5}\right)^{1}$

$-\frac{9}{5}$

$x = 0$

$-3\left(\frac{5}{3}\right)^{0}$

$-3(1)$

-3

$x = 1$

$-3\left(\frac{5}{3}\right)^{1}$

$-\cancel{3}\left(\frac{5}{\cancel{3}}\right)$

-5

$x = 2$

$-3\left(\frac{5}{3}\right)^{2}$

$-\cancel{3}\,\frac{25}{3^{\cancel{2}}}$

$-\frac{25}{3}$

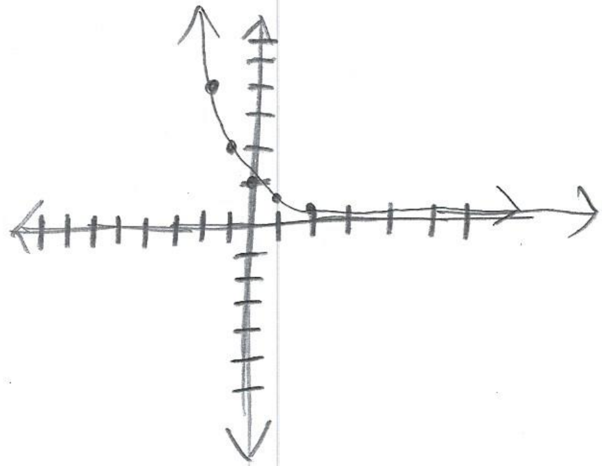
Horizontal Asymptote at $y = 0$.

Domain: $(-\infty, \infty)$

Range: $(-\infty, 0)$

Ex: $f(x) = 2^{-x}$

x	y
-2	$2^{-(-2)} = 2^2 = 4$
-1	$2^{-(-1)} = 2^1 = 2$
0	$2^{-0} = 1$
1	$2^{-(1)} = \frac{1}{2}$
2	$2^{-(2)} = \frac{1}{2^2} = \frac{1}{4}$



Domain: $(-\infty, \infty)$
Range: $(0, \infty)$

Ex: $f(x) = \left(\frac{1}{4}\right)^x - 1$

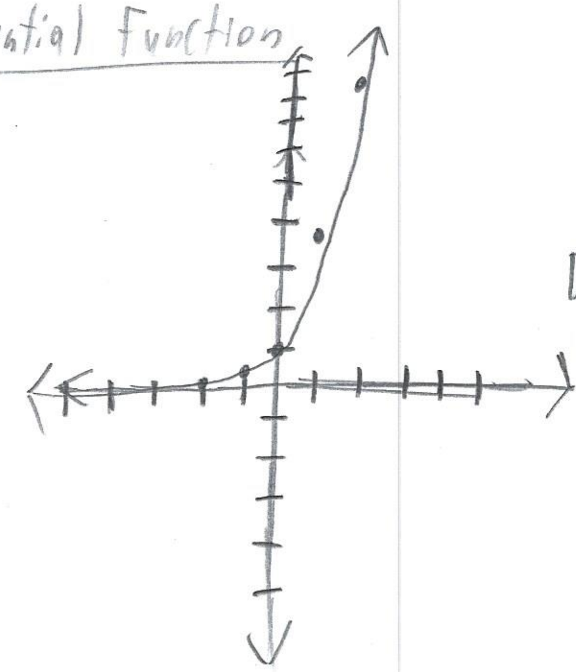
Domain: $(-\infty, \infty)$
Range: $(-1, \infty)$

The natural Exponential Function

$e \approx 2.718$

$y = e^x$

x	y
-2	$e^{-2} = \frac{1}{e^2} \approx 0.14$
-1	$e^{-1} = \frac{1}{e} \approx 0.37$
0	$e^0 = 1$
1	$e^1 \approx 2.72$
2	$e^2 \approx 7.39$



Horz Asymp. at $y = 0$.

Domain: $(-\infty, \infty)$
Range: $(0, \infty)$

Word problem from Aleks

pricipal amount is \$4,500 invested with an intrest rate of 6.75% and is compounded annually. How much is the investment worth after 12 years?

The intrest formula is

$$\underset{\substack{\uparrow \\ \text{principal}}}{a}\,(1+\underset{\substack{\uparrow \\ \text{intrest} \\ \text{rate in} \\ \text{decimal}}}{r})^{t \leftarrow \text{time}}$$

So for our question plug in the info:

$$4500\,(1+0.0675)^{12} =$$

$$= 4500\,(1.0675)^{12}$$

$$= \$9854.33 \approx \$9854$$

Logarithms

$\log$ ← common logarithm

$\ln$ ← natural logarithm

Defenition of logarithmic function

If x and b are positive real numbers such that $b \neq 1$, then $y = \log_b(x)$ is called a logarithmic function with base b.

$y = \log_b(x)$ is equivalent to $b^y = x$.

Find the inverse of $f(x) = b^x$

$$y = b^x$$
$$x = b^y$$
$$y = \log_b(x)$$
$$f^{-1}(x) = \log_b(x)$$

So we can see that exponential functions and logarithmic functions are inverses of each other.

Ex from Aleks:

1) Convert to exponential notation.

$$\log_3\left(\frac{1}{81}\right) = -4$$

$$3^{-4} = \frac{1}{81}$$

general form

$$\log_b(x) = y$$

$$b^y = x$$

2) $\ln 9 = y$

$$e^y = 9$$

$$\ln_e(x) = y$$

$$e^y = x$$

$\ln(x)$ has an implied base of e, $\ln_e(x)$

3) $\log_8(64) = \boxed{2}$

$$8^{\underline{\ \ }} = 64$$

$$8^2 = 64$$

4) $\log_4(64) = \boxed{3}$

$$4^{\underline{\ \ }} = 64$$

$$4^3 = 64$$

5) $\log_3\left(\frac{1}{27}\right) = \boxed{-3}$

$$3^{\underline{\ \ }} = \frac{1}{27}$$

$$3^{-3} = \frac{1}{27}$$

Logarithm Packet

Ex:

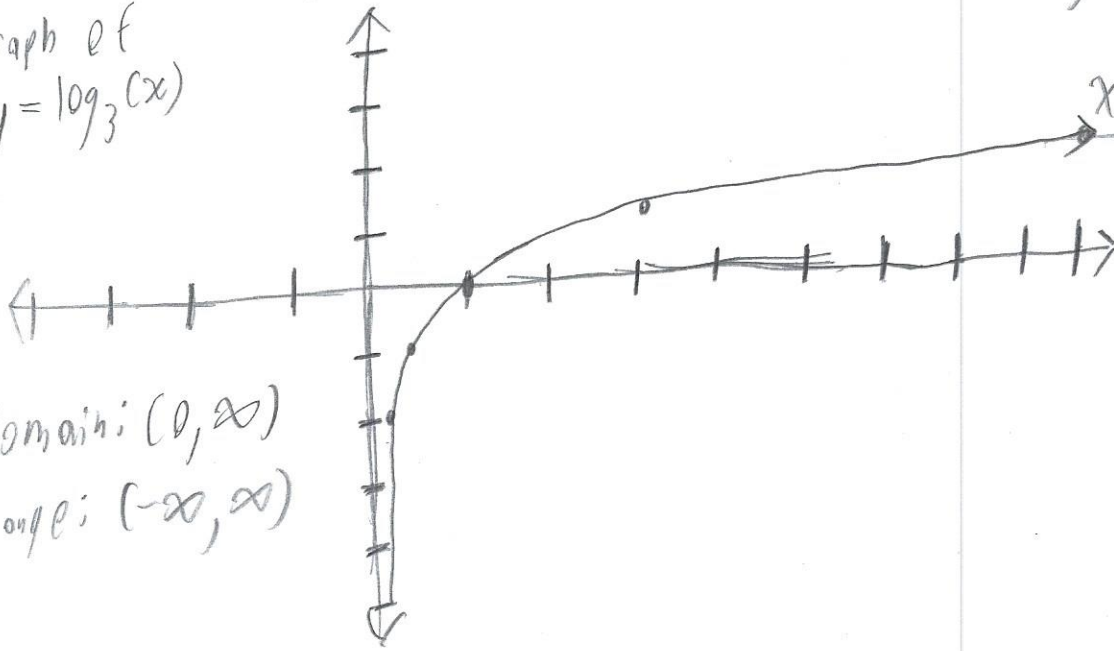
right 3 up 4

$y = \log_3(x-3) + 4$

$y = \log_3(x)$

$3^y = x$

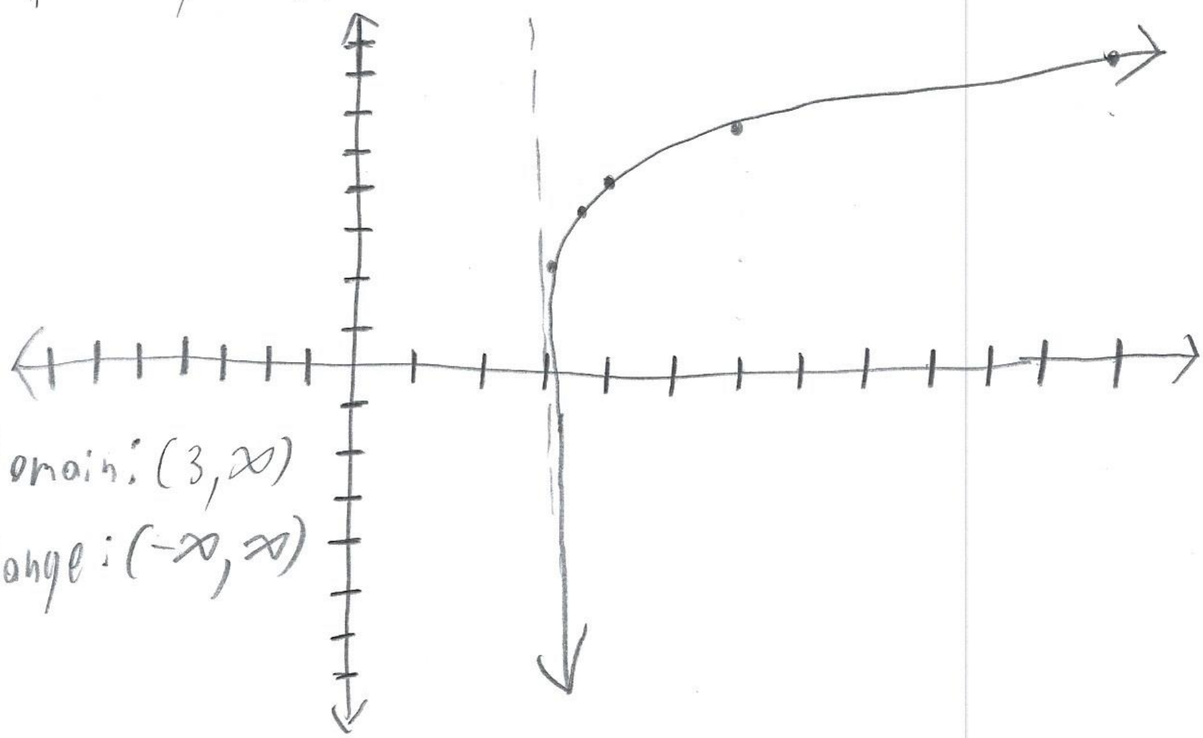
graph of
$y = \log_3(x)$



Domain: $(0, \infty)$
Range: $(-\infty, \infty)$

$x = 3^y$	y
$\frac{1}{9}$	-2
$\frac{1}{3}$	-1
1	0
3	1
9	2

translated
graph of $y = \log_3(x-3) + 4$



Domain: $(3, \infty)$
Range: $(-\infty, \infty)$