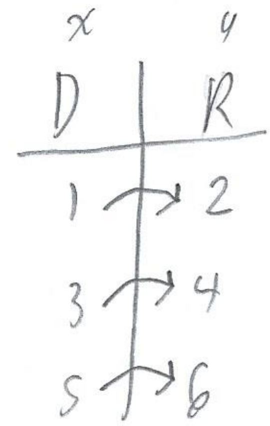
6/3

Relations

A set of ordered pairs

$\{(1,2),(3,4),(5,6)\}$



Function

A special type of relation in which for each x-value there is one y-value which it is mapped to.

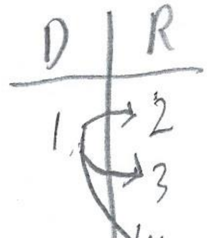
Domain

The set of all x-values for which your function is defined.

Range

The set of all y-values for which your function is defined.

Is the relation $\{(1,2)(1,3),(1,4)\}$ a function?

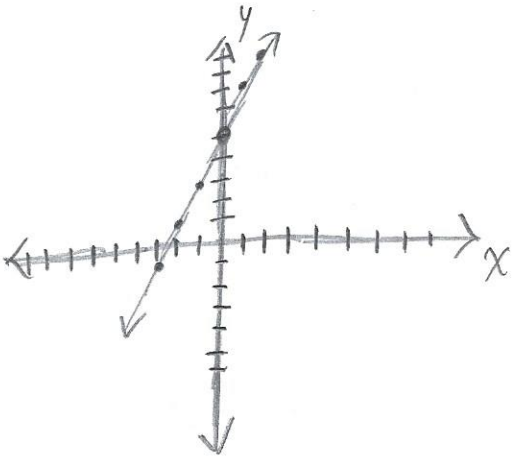


Not a function because the x-value 1 has several y-values.

Ex:

$y = 2x + 5$ is this a function?

This will be a line since it follows the form $y = mx + b$.



y-intercept $(0, 5)$

x-intercept $(-2.5, 0)$

We can see from sketching the graph that it will pass the vertical line test, thus this equation is a function.

Ex:

$x^2 + y^2 = 4$

Center radius form of a circle: $(x-h)^2 + (y-k)^2 = r^2$

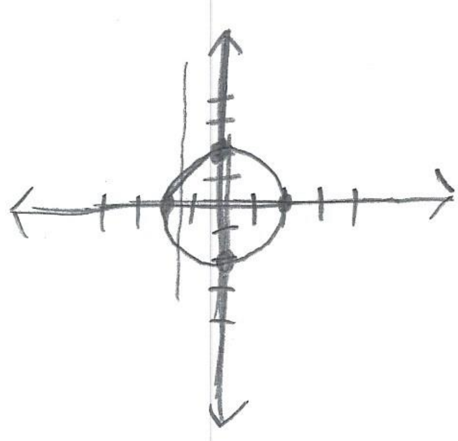
center: (h, k)

radius $= r$

$$(\underset{\substack{\uparrow \\ h}}{x - 0})^2 + (\underset{\substack{\uparrow \\ k}}{y - 0})^2 = 2^2$$

center: $(0,0)$

radius $= 2$



Fails the vertical line test so this is not a function

One-to-one:

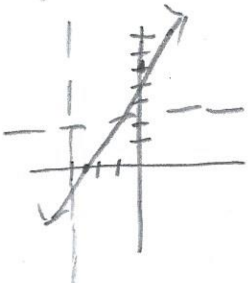
A relation is one-to-one if for each y-value there is exactly one x-value.

Vertical line test: tells whether or not something is a function.

Horizantal line test: tells whether or not something is one-to-one.

Ex:

$y = 2x + 5$

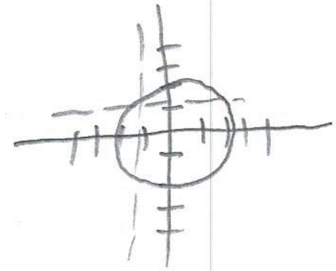


Vert Test:
Is a function

Horz Test:
Is one-to-one

Ex:

$x^2 + y^2 = 4$



Vert Test:
Not a function

Horz Test:
Not one-to-one

Book defenition of one-to-one:

A function f is a one-to-one function, if for any a and b in the domain of f, if $a \neq b$, then $f(a) \neq f(b)$. Equivalently if $f(a) = f(b)$ then $a = b$.

Ex: $f(x) = 2x + 5$

name of function

variable of function

$$f(a) = f(b)$$
$$2a + 5 = 2b + 5$$
$$-5 \qquad -5$$
$$\frac{2a}{2} = \frac{2b}{2}$$
$$a = b$$

Ex: Is $f(x) = x^2 + 1$ a one-to-one function?

If $f(a) = f(b)$

$$a^2 + 1 = b^2 + 1$$
$$-1 \qquad -1$$

$$a^2 = b^2$$

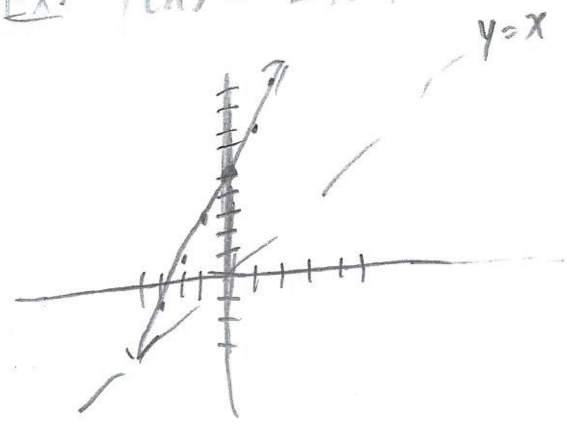
$$a = \pm b$$

So since a does not equal only b the $f(x) = x^2 + 1$ is not a one-to-one function.

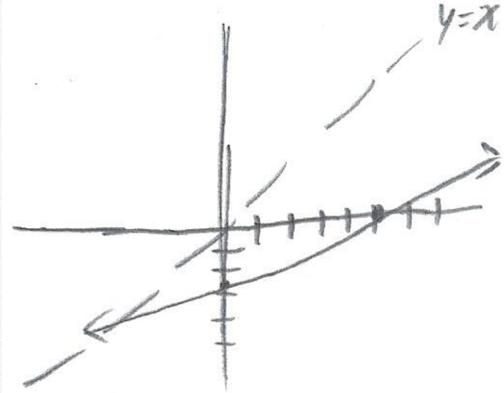
3.1 Inverse Functions:

Before you find the inverse of a function you need to verify that the function is a function and that it is specifically a one-to-one function.

Ex: $f(x) = 2x + 5$



The inverse of a graph is just its reflection over the line $y = x$.



To find the inverse algebraically

1) Replace the $f(x)$ with y
2) switch the x and y
3) Solve for y.
4) Replace y with $f^{-1}(x)$

↑ said f inverse of x.

$$f(x) = 2x + 5$$
$$y = 2x + 5$$
$$x = 2y + 5$$
$$x - 5 = 2y$$
$$y = \frac{x-5}{2}$$
$$f^{-1}(x) = \frac{x-5}{2}$$

To verify that the inverse you got is actually an inverse check if $f(f^{-1}(x)) = x$, and $f^{-1}(f(x)) = x$.

Ex:
$$f(x) = 2x + 5$$
$$f^{-1}(x) = \frac{x-5}{2}$$

Does $f(f^{-1}(x)) = x$?

$$2\left(\frac{x-5}{2}\right) + 5$$
$$\cancel{2} \cdot \frac{x-5}{\cancel{2}} + 5$$
$$x \cancel{-5} \cancel{+5}$$
$$x = f(f^{-1}(x)) \checkmark$$

Does $f^{-1}(f(x)) = x$?

$$\frac{(2x+5)-5}{2}$$
$$\frac{2x \cancel{+5} \cancel{-5}}{2}$$
$$\frac{\cancel{2}x}{\cancel{2}}$$
$$x = f^{-1}(f(x)) \checkmark$$

Ex:
$$f(x) = -\frac{x}{3} \qquad g(x) = -3x$$

$$f(g(x)) = \cancel{-}\frac{+3x}{3}$$
$$= +\frac{\cancel{3}x}{\cancel{3}}$$
$$= x$$

$$g(f(x)) = -3\left(-\frac{x}{3}\right)$$
$$= +3 \cdot +\frac{x}{3}$$
$$= +3 \cdot \frac{x}{3}$$
$$= \frac{\cancel{3}x}{\cancel{3}} = x$$

So $f(x)$ and $g(x)$ are inverses of each other.