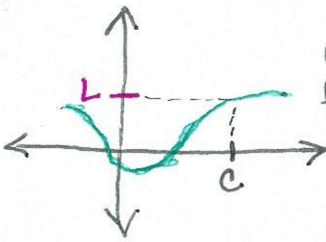
Limits

– the function f has a Limit L as x approaches a real number C, written as $\lim_{x \to C} f(x) = L$



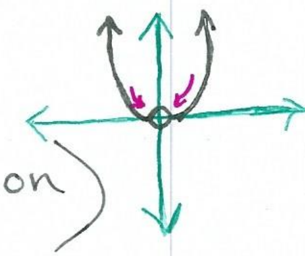
where is the function going as I approach it.

ex

$$\lim_{x \to 0} \frac{x^3}{x}$$

$\frac{x^3}{x} = x^2$

but $x \neq 0$



"Looks like its going to zero as I come from Left and right."

The Limit can exist at C even if the function is not defined at C.

done by Graphing.

$$\boxed{\lim_{x \to 0} \frac{x^3}{x} = 0}$$

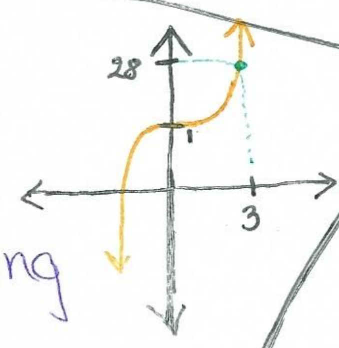
**Now Numerical (Table)

x	-1	$-\frac{1}{10}$	$-\frac{1}{100}$	0	$\frac{1}{100}$	$\frac{1}{10}$	1
$f(x)$	1	$\frac{1}{100}$	$\frac{1}{10000}$	u	$\frac{1}{1000}$	$\frac{1}{100}$	1

we know for a fact that x^2 will always give you a positive $f(x)$. It wont be a negative number and it get close to zero.

ex 2

$$\lim_{x \to 3} x^3 + 1$$



$3^3 + 1 = 28$

no issue w/plugging in the 3 into our function. Unlike the first example.

$$\boxed{\lim_{x \to 3} x^3 + 1 = 28}$$

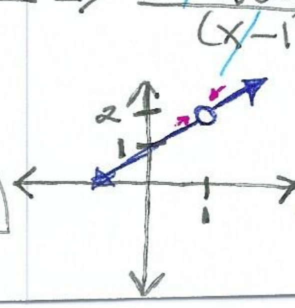
ex3

$$\lim_{x \to 1} \frac{x^2 - 1}{x - 1}$$

Here we have a Problem if we let $x = 1$ then we get 0 in the denominator.

$$\frac{x^2-1}{x-1} \Rightarrow \frac{\cancel{(x-1)}(x+1)}{\cancel{(x-1)}} \Rightarrow x+1$$

$$\boxed{\lim_{x \to 1} \frac{x^2-1}{x-1} = 2}$$



It looks like it is going to 2 from both sides.

Properties of Limits

Let $\lim_{x \to c} f(x) = L$ and $\lim_{x \to c} g(x) = M$

1. $\lim_{x \to c} (f(x))^r = (\lim_{x \to c} f(x))^r = L^r$ • "r" is a positive number

2. $\lim_{x \to c} K \cdot f(x) = K(\lim_{x \to c} f(x)) = K \cdot L$ • "k" is a constant

3. $\lim_{x \to c} [f(x) \pm g(x)] = \lim_{x \to c} f(x) \pm \lim_{x \to c} g(x) = L \pm M$

4. $\lim_{x \to c} [f(x)\, g(x)] = \lim_{x \to c} f(x) \cdot \lim_{x \to c} g(x) = L \cdot M$

5. $\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{\lim_{x \to c} f(x)}{\lim_{x \to c} g(x)} = \frac{L}{M}$, where $M \neq 0$

Problem 1

$$\lim_{x \to 2} \frac{x+1}{x+2} = \frac{\lim_{x \to 2} x+1}{\lim_{x \to 2} x+2} = \frac{2+1}{2+2} = \frac{3}{4}$$

Problem 2

$$\lim_{x \to 0} x^2 + x = \lim_{x \to 0} x^2 + \lim_{x \to 0} x = 0^2 + 0 = 0$$

Problem 3

$$\lim_{x \to 4} (x+1)^3 = (\lim_{x \to 4} (x+1))^3 = (4+1)^3 = 5^3 = 125$$

Indeterminate form

"$\frac{0}{0}$", "$\frac{\infty}{\infty}$", "0^0", "1^∞"

ex $\lim_{x \to 2} \frac{4(x^2-4)}{x-2}$

If you plug 2 into the function you get $\frac{0}{0}$. So reduce Algebraiclly.

$$\frac{4(x^2-4)}{x-2} = \frac{4 \cdot (x+2)\cancel{(x-2)}}{\cancel{(x-2)}} = 4(x+2)$$

$$\boxed{\lim_{x \to 2} 4(x+2) = 4(2+2) = 16}$$