Review Day for final

Section 1.) Solve initial value problems

(a) $f'(x)=3x^2+4x-1$: $f(2)=9$

$\int 3x^2+4x-1\,dx = x^3+2x^2-x+C$

$9=2^3+2\cdot 2^2-2+C$

$9=8+8-2+C$

$9=14+C$ $\quad C=-5$ $\quad \boxed{f(x)=x^3+2x-x-5}$

(b) $f'(x)=1+\frac{1}{x^2}$; $f(1)=3$

$\int 1+\frac{1}{x^2}\,dx = x-\frac{1}{x}+C$

$3=1-\frac{1}{1}+C$

$3=0+C$ $\quad C=3$

$\boxed{f(x)=x-\frac{1}{x}+3}$

(C) $f'(x)=\frac{1}{\sqrt{x}}$; $f(4)=2$ $\quad \int \frac{1}{\sqrt{x}}dx = \frac{\sqrt{x}}{2}+C$

$2=\frac{\sqrt{4}}{2}+C$

$2=1+C$

$C=1$

$\boxed{f(x)=\frac{\sqrt{x}}{2}+1}$

(D) $f''(x)=3x$; $f'(3)=1$; $f(0)=4$

$\int 3x\,dx = \frac{3x^2}{2}+C$ $\quad 1=\frac{3\cdot 3^2}{2}+C$ $\quad 1=\frac{27}{2}+C$ $\quad C=1-\frac{27}{2}$

$f'(x)=\frac{3x^2}{2}+1-\frac{27}{2}$

$\int \frac{3x^2}{2}+1-\frac{27}{2}\,dx = \frac{x^3}{2}+\left(1-\frac{27}{2}\right)x+C$ $\quad 4=\frac{0^3}{2}+\left(1-\frac{27}{2}\right)\cdot 0+C$

$4=C$

$\boxed{f(x)=\frac{x^3}{2}+\left(1-\frac{27}{2}\right)x+4}$

$f(x)=\frac{x^3}{2}-\frac{25}{2}x+4$

Section 2: Evaluate the following definite integrals.

(a) $\int_0^1 x^3+x\,dx = \frac{x^4}{4}+\frac{x^2}{2}\Big|_0^1$

$\frac{1^4}{4}+\frac{1^2}{2}-\left[\frac{0^4}{4}+\frac{0^2}{2}\right] = \left[\frac{1}{4}+\frac{1}{2}\right]$ $\frac{1}{4}+\frac{2}{4}=\frac{3}{4}$

(b) $\int_{-1}^2 x^2-2x+2\,dx = \frac{x^3}{3}-\frac{2x^2}{2}+2x\Big|_{-1}^2$

$\frac{2^3}{3}-2^2+2\cdot 2-\left[\frac{(-1)^3}{3}-(-1)^2+2(-1)\right] = \frac{8}{3}+\frac{1}{3}-3 = \frac{9}{3}-\frac{3}{1}=3-3=0$

(c) $\int_0^4 2+\sqrt{x}\,dx = 2x+\frac{2x^{3/2}}{3}\Big|_0^4$

$2(4)+\frac{2(4)^{3/2}}{3}-\left(2(0)+\frac{2(0)^{3/2}}{3}\right) = \frac{8}{1}+\frac{16}{3}=\frac{24}{3}+\frac{16}{3}=\frac{40}{3}$

Section 3: Word Problems:

1) $R(x)=P\cdot x = x(-0.04x+800) = -0.04x^2+800x$

$R'(x) = -0.08x+800$

$P(x)=R(x)-C(x) = -0.04x^2+800x-200x-30000 = -0.04x^2+600x-30000$

$P'(x) = -0.08x+600$

$C'(x) = 200$

$R'(5000) = -0.08(5000)+800 = 400$

$P'(5000) = -0.08(5000)+600 = 200$

$C'(5000) = 200$