

Definite Integral: represents the area underneath a function to the x-axis between two points.

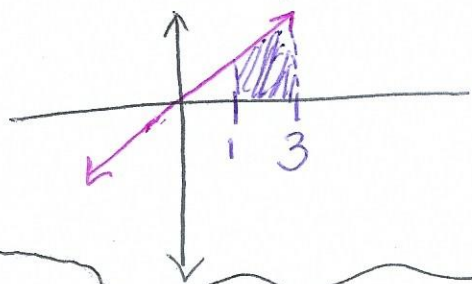
Notation:

$$\int_a^b f(x) dx$$

Let $F(x)$ be the antiderivative of $f(x)$.

$$F(x) \Big|_a^b = F(b) - F(a)$$

visual: $f(x) = x$



(ex) $\int_1^3 x dx = \frac{x^2}{2} \Big|_1^3 = \frac{3^2}{2} - \frac{1^2}{2} = \frac{9-1}{2} = \frac{8}{2} = 4$

4 area under the function between the bounds.

$$1.) \int_2^3 x^2 + 3x dx = \frac{x^3}{3} + \frac{3x^2}{2} \Big|_2^3$$

$$\frac{3^3}{3} + \frac{3(3)^2}{2} - \left[\frac{2^3}{3} + \frac{3(2)^2}{2} \right] = \frac{27}{3} + \frac{27}{2} - \frac{8}{3} - \frac{12}{2} = \frac{19}{3} + \frac{15}{2}$$

$$\frac{38}{6} + \frac{45}{6} = \boxed{\frac{83}{6}} = \boxed{13\frac{5}{6}}$$

$$2.) \int_4^9 \sqrt{x} dx = \frac{2x^{3/2}}{3} \Big|_4^9 = \frac{2(9)^{3/2}}{3} - \frac{2(4)^{3/2}}{3} = \frac{2 \cdot 3^3}{3} - \frac{2 \cdot 2^3}{3} = \frac{18}{1} - \frac{16}{3} = \frac{54}{3} - \frac{16}{3} = \boxed{\frac{38}{3}}$$

$$3) \int_1^2 3x^2 + 4x - 1 \, dx = x^3 + 2x^2 - x \Big|_1^2$$

$$2^3 + 2 \cdot 2^2 - 2 - [1^3 + 2(1)^2 - 1] = 8 + 8 - 2 - 1 - 2 + 1 = \boxed{12}$$

$$4) f''(x) = 10, \quad f'(1) = 7, \quad f(2) = 12, \quad f(x) = ?$$

$$\int 10 \, dx = 10x + C$$

$$f'(x) = 10x - 3$$

$$7 = 10(1) + C$$

$$-3 = C$$

$$\int 10x - 3 = 5x^2 - 3x + C$$

$$12 = 5(2)^2 - 3(2) + C$$

$$12 = 5(4) - 6 + C$$

$$12 = 20 - 6 + C$$

$$12 = 14 + C$$

$$-2 = C$$

$$f(x) = 5x^2 - 3x - 2$$

$$5) \int_2^4 x^2 - 2x + 4 \, dx = \frac{x^3}{3} - x^2 + 4x \Big|_2^4$$

$$\frac{4^3}{3} - 4^2 + 4 \cdot 4 - \left(\frac{2^3}{3} + 2^2 - 4 \cdot 2 \right)$$

$$\frac{64}{3} - \cancel{16} + \cancel{16} - \left(\frac{8}{3} + 4 - 8 \right)$$

$$\frac{56}{3} - \frac{4}{1}$$

$$\frac{56}{3} - \frac{12}{3} = \boxed{\frac{44}{3}}$$