Solve Initial value problem

1.) $f'(x) = 4x+2$, $f(1) = 4$ find $f(x)$.

$\int 4x+2\,dx \Rightarrow 4\int x\,dx + 2\int dx \Rightarrow 4\left(\frac{x^2}{2}\right)+2(x) = 2x^2+2x$

$= 2(x^2+x)+C$

$4 = 2(1^2+1)+C$ $\quad 4 = 4+C$

$4 = 2(2)+C$ $\quad 0 = C$

$$f(x) = 2(x^2+x)$$

2.) $g'(x) = 7x^2+e^x$ $f(0) = 5$ find $g(x)$.

$\int 7x^2+e^x\,dx \Rightarrow 7\int x^2_{dx} + \int e^x dx \Rightarrow 7\left(\frac{x^3}{3}\right)+e^x+C$

$5 = 7\left(\frac{0^3}{3}\right)+e^0+C = 1+C$ $\quad C = 4$

$$g(x) = \frac{7}{3}x^3+e^x+4$$

3.) $f''(x) = 3x+2$, $f'(2) = 3$, $f(0) = 1$ Find $f(x)$.

$\int 3x+2\,dx \Rightarrow 3\int x\,dx + 2\int dx \Rightarrow \frac{3}{2}x^2+2x+C$

$3 = \frac{3}{2}(2^2)+2(2)+C$

$3 = 6+4+C$ $\quad f'(x) = \frac{3}{2}x^2+2x-7$

$3 = 10+C$

$-7 = C$

$\frac{3}{2}\int x^2_{dx} + 2\int x\,dx - 7\int dx \Rightarrow \frac{x^3}{2}+x^2-7x+C$

$1 = \frac{0^3}{2}+0^2-7(0)+C$

$C = 1$

$$f(x) = \frac{x^3}{2}+x^2-7x+1$$

4.) $g''(x) = 7$, $g'(10) = 8$, $g(7) = 12$

find $g(x)$.

$7\int dx \Rightarrow 7x + C$ $\quad 8 = 7(10) + C$ $\quad 8 = 70 + C$ $\quad C = -62$

$g'(x) = 7x - 62$ $\quad 7\int x\,dx - 62\int dx \Rightarrow$

$\frac{7}{2}x^2 - 62x + C$

$12 = \frac{7}{2}(7^2) - 62(7) + C$

$12 = -262.5 + C$

$274.5 = C$

$$g(x) = \frac{7}{2}x^2 - 62x + 274.5$$