Antiderivatives

Recall: Derivatives $\frac{d}{dx} f(x) = f'(x)$

$$\underbrace{\int f'(x)\,dx}_{\text{Indefinite integral}} = f(x)$$

Recall: $f'(x) = 2x$

$f(x) = x^2$ $\boxed{}$

we don't know if there was a constant before we took the derivative.

- So when we do indefinite integral we always write "+C" next to the function

Constant Rule

$$\frac{d}{dx}(kx) = k$$

$$\int k\,dx = kx + C$$

when doing antiderivative it should bring you back to before you took the derivative.

Power Rule

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\int x^n = \frac{x^{n+1}}{n+1} + C$$

or

$$\frac{1}{n+1} \cdot x^{n+1} + C$$

$\boxed{ex}$ $\int x^3\,dx = \frac{x^4}{4} + C$

Constant Multiple Rule

$$\frac{d}{dx}(k(f(x))) = k \cdot f'(x)$$

$$\int k \cdot f'(x)\,dx = k\int f'(x)\,dx$$

$$= k \cdot f(x) + C$$

Sum + Difference Rule

$$\frac{d}{dx}(f(x) \pm g(x)) = \frac{d}{dx}f(x) \pm \frac{d}{dx}g(x) = f'(x) \pm g'(x)$$

$$\int (f'(x) \pm g'(x))dx = \int f'(x)dx \pm \int g'(x)dx = f(x) + g(x) + C$$

$\boxed{ex}$ $\int (3x^2 + 4x)dx = 3\int x^2 dx + 4\int x\,dx =$

$3 \cdot \frac{x^3}{3} + C_1 + 4 \cdot \frac{x^2}{2} + C_2 = x^3 + 2x^2 + C_1 + C_2$

Special Case: $\int \frac{1}{x}dx = \ln|x| + C$

Practice Problems

1.) $\int 15x^2 + e^x dx$

$15\int x^2 dx + \int e^x dx$

$15\left[\frac{x^3}{3}\right] + C_1 + e^x + C_2$

$5x^3 + C_1 + e^x + C_2$

$\boxed{5x^3 + e^x + C_1 + C_2}$

2.) $\int e + 2x - \frac{1}{x} dx$

$e\int dx + 2\int x dx - \int \frac{1}{x} dx$

$xe + C_1 + 2\frac{x^2}{2} + C_2 - \ln|x| + C_3$

$\boxed{ex + x^2 - \ln|x| + C_1 + C_2 + C_3}$

3.) $\int \frac{2}{x} + \frac{5}{x^3} dx$

$2\int \frac{1}{x}dx + 5\int \frac{1}{x^3}dx$

$2(\ln|x| + C_1) + 5\int x^{-3}dx$

$2\ln|x| + C_1 + 5\left(\frac{x^{-2}}{-2}\right) + C_2$

$2\ln|x| - \frac{5}{2}x^{-2} + C_1 + C_2$

$\boxed{2\ln|x| - \frac{5}{2x^2} + C_1 + C_2}$

4.) $\int \frac{1}{3\sqrt{x}}dx = \frac{1}{3}\int \frac{1}{\sqrt{x}}dx = \frac{1}{3}\int (x)^{-1/2}dx$

$= \frac{1}{3}\left(\frac{x^{1/2}}{1/2}\right) + C = \frac{2}{3}\sqrt{x} + C = \boxed{\frac{2\sqrt{x}}{3} + C}$

- When you have multiple constants you can condense them into one "C."

Answers to the practice problems

1.) $5x^3 + e^x + C$

2.) $ex + x^2 - \ln|x| + C$

3.) $2\ln|x| - \frac{5}{2x^2} + C$

4.) $\frac{2\sqrt{x}}{3} + C$

These are called General Solution!!

Initial Value Problem

- Find a particular solution

$f'(x) = 2x + \frac{1}{x}$ $\quad f(1) = 5.$

$\int 2x + \frac{1}{x}\,dx$

$2\int x\,dx + \int \frac{1}{x}\,dx$

$x^2 + \ln|x| + C = f(x)$

General Solution

But we were given more info. Use it.

$(1)^2 + \ln|1| + C = 5$

$1 + 0 + C = 5$

$1 + C = 5$

$C = 4$

$x^2 + \ln|x| + 4 = f(x)$

Particular Solution

Practice Problems

(1)

$f'(x) = 2 + e^x$ $\quad f(2) = 12$

Find $f(x)$

$\int 2 + e^x\,dx = 2\int dx + \int e^x\,dx$

$= 2x + e^x + C$

$2(2) + e^2 + C = 12$

$4 + e^2 + C = 12$

$C = 8 - e^2$

$\boxed{f(x) = 2x + e^x + 8 - e^2}$

(2) $f'(t) = 4 + 5t^{2/3}$ $\quad f(0) = 3000$

$\int 4 + 5t^{2/3}\,dt$

$4\int dt + 5\int t^{2/3}\,dt$

$4t + \frac{5t^{5/3}}{5/3} + C$

$4t + 3t^{5/3} + C = f(x)$

$4(0) + 3(0)^{5/3} + C = 3000$

$\boxed{4t + 3t^{5/3} + 3000 = f(x)}$