Implicit Differentiation

$y = f(x)$ $\quad y = x^2+3x+2$ $\quad x = g(t)$

$y' = f'(x)$ $\quad y' = 2x+3$ $\quad y = f(t)$

Recall:

Chain Rule

$$\frac{d}{dx}(f(g(x))) = \frac{d}{dx}f'(g(x)) \cdot g'(x)$$

$$\frac{d}{dx}((f(x))^2) = 2(f(x)) \cdot f'(x)$$

$$\frac{d}{dx}y^2 = 2(y) \cdot y'$$

$$\frac{d}{dt} \quad x^2+y^2 = 9$$

$$\frac{d}{dt}(x^2) + \frac{d}{dt}(y^2) = \frac{d}{dt}(9)$$

$$2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt} = 0$$

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example $x = \dfrac{4x^2+3}{x+y^2}$ Find $\dfrac{dy}{dx} = y'$

$$x^2 + y^2 x = 4x^2 + 3$$

$$y^2 x = 4x^2 - x^2 + 3 = 3x^2 + 3$$

$$y^2 = \frac{3x^2+3}{x} = 3x + 3/x$$

$y^2 = 3x + 3/x$ $\quad \rightarrow \quad y = \sqrt{3x+3/x}$

$2y \cdot y' = 3 + -3/x^2$

$$y' = \frac{3 - 3/x^2}{2y} \rightarrow y' = \frac{3 - 3/x^2}{2(\sqrt{3x+3/x})}$$