- Higher Order deratives

Distance - $f(x)$

Velocity - $f'(x)$

Accelaration - $f''(x)$

(ex) Find the first, second, and third derivatives.

$$f(x) = 8x^3 + 4x^2 - 7x + 2$$

$$f'(x) = 24x^2 + 8x - 7$$

$$f''(x) = 48x + 8$$

$$f'''(x) = 48 \quad \text{or } f^{(3)}(x) = 48$$

$f^{(n)}(x)$ to denote derivative 4 or greater.

(ex2) $g(x) = \ln(x^2+1)$

$$g'(x) = \frac{1}{x^2+1} \cdot 2x = \frac{2x}{x^2+1}$$

$$g''(x) = \frac{2(x^2+1) - (2x)(2x)}{(x^2+1)^2} = \frac{2x^2+2-4x^2}{(x^2+1)^2} = \frac{-2x^2+2}{(x^2+1)^2} = \frac{2-2x^2}{x^4+2x^2+1}$$

$$g'''(x) = \frac{(-4)(x^4+2x^2+1) - (4x^3+4x)(2-2x^2)}{(x^4+2x^2+1)^2} = \frac{-4x^4-8x^2-4-8x^3+8x^5-8x+8x^3}{(x^4+2x^2+1)^2}$$

$$= \frac{8x^5-4x^4-8x^2-8x-4}{(x^4+2x^2+1)^2}$$

Recall: Indetterminate forms

$\boxed{\text{“}\frac{0}{0}\text{”} \ \text{“}\frac{\infty}{\infty}\text{”}}$ “$0 \cdot \infty$” “1^∞”

“$\infty \cdot \infty$” “∞^0”

If we can have the form “$\frac{0}{0}$” “$\frac{\infty}{\infty}$” we use L'Hôspital's Rule

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

$\boxed{\text{ex}}$ $\displaystyle \lim_{x \to 5} \frac{x^2-25}{x^2-10x+5} \overset{L'H}{=} \text{“}\frac{0}{0}\text{”} \lim_{x \to 5} \frac{2x}{2x-10} = \frac{2(5)}{2(5)-10} = \frac{10}{10-10}$

$= \frac{10}{0}$ The limit DNE

$\boxed{\text{ex2}}$ $\displaystyle \lim_{x \to \infty} \frac{x^2+e^x}{x^3} \overset{L'H \text{ “}\frac{\infty}{\infty}\text{”}}{=} \lim_{x \to \infty} \frac{2x+e^x}{3x^2} \overset{L'H \text{ “}\frac{\infty}{\infty}\text{”}}{=}$

$$\lim_{x \to \infty} \frac{2+e^x}{6x} \overset{L'H \text{ “}\frac{\infty}{\infty}\text{”}}{=} \lim_{x \to \infty} \frac{e^x}{6} = \text{DNE}$$

$$\lim_{x\to 0} x\ln(x) \Rightarrow \lim_{x\to 0} \frac{\ln(x)}{\frac{1}{x}} \overset{L'H}{=}^{``\frac{\infty}{\infty}"} \lim_{x\to 0} \frac{\frac{1}{x}}{\frac{-1}{x^2}} \Rightarrow \lim_{x\to 0} -\frac{x^2}{x} \rightarrow \lim_{x\to 0} -x$$

$$= \boxed{0}$$