Functions:

Exponential and Logarithms

- **Composition of functions** - two functions f and g, f composed with g is written as $(f \circ g)(x)$ or $(f(g(x)))$

$$x \rightarrow \boxed{g} \rightarrow \boxed{g(x)} \rightarrow \boxed{f} \rightarrow \boxed{f(x)}$$

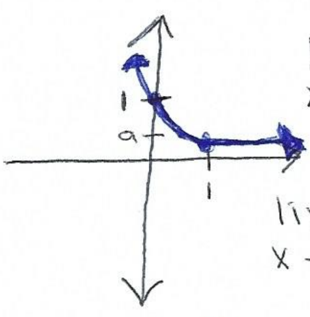
ex $f(x) = 4x^2$ $g(x) = \frac{1}{x}$

$f(g(x)) = 4\left(\frac{1}{x}\right)^2 = 4\left(\frac{1}{x^2}\right) = \frac{4}{x^2}$

- **Inverse functions** - two functions are inverses of each other if and only if $f(g(x)) = x$ and $g(f(x)) = x$.

- **Exponentials** - $\cdot\, a^x$ $a>0, a \neq 1$

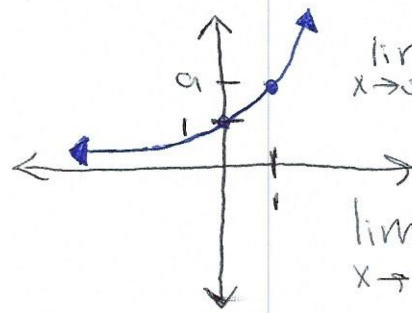
when $0<a<1$



$\lim_{x \to \infty} a^x = 0$

$\lim_{x \to -\infty} a^x = DNE$

when $a>1$



$\lim_{x \to \infty} a^x = DNE$

$\lim_{x \to -\infty} a^x = 0$

Exponent Rules

* $a^x \cdot a^y = a^{x+y}$
* $a^0 = 1$
* $\frac{a^x}{a^y} = a^{x-y}$
* $a^1 = a$
* $(a^x)^y = a^{xy}$
* $a^{-x} = \frac{1}{a^x}$

write in simplest terms

1.) $(15^{2x})(15^{x^2})$
$= 15^{2x+x^2}$

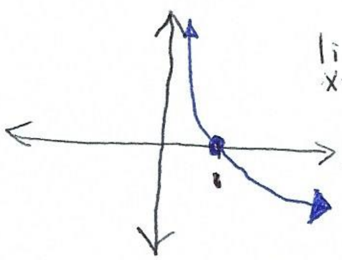
2.) $\frac{(2^{2x})^x}{2^{2x^2}}$
$= \frac{2^{2x^2}}{2^{2x^2}} = 2^0 = 1$

3.) $(2^{-x})^{3x^2}$
$= 2^{-3x^3}$
$= \boxed{\frac{1}{2^{3x^3}}}$

Logarithms

- $\log_a(x)$
  - inverse of a^x, so $y = \log_a(x) \Longleftrightarrow a^y = x$
  - $\log_a(a^x) = a^{\log_a(x)} = x$
  - $x > 0$

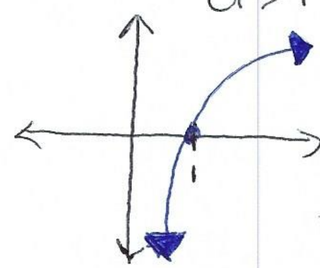
$0 < a < 1$



$\lim_{x\to\infty} \log_a(x) = \text{DNE}$

$\lim_{x\to-\infty} \log_a(x) = \text{DNE}$

$a > 1$



$\lim_{x\to\infty} \log_a(x) = \text{DNE}$

$\lim_{x\to-\infty} \log_a(x) = \text{DNE}$

Logarithm Rules

- $\log_a(xy) = \log_a(x) + \log_a(y)$
- $\log_a\left(\frac{x}{y}\right) = \log_a(x) - \log_a(y)$
- $\log_a(x^y) = y\log_a(x)$
- $\log_a\left(\frac{1}{x}\right) = -\log_a(x)$
- $\log_a(1) = 0$
- $\log_a(a) = 1$
- $\log_a(a^k) = k$

ex $\log_2(15x^2)$

$\log_2(15) + \log_2(x^2)$

$\boxed{\log_2(15) + 2\log_2(x)}$

ex $\log_2(16) \Rightarrow \log_2(2^4) \Rightarrow 4\log_2(2) \Rightarrow 4\cdot 1 = 4$

1.) $\log_2(15x^2 \cdot 2^x)$

$\log_2(15x^2) + \log_2(2^x)$

$\log_2(15) + 2\log_2(x) + x\log_2(2)$

$\boxed{\log_2(15) + 2\log_2(x) + x}$

2.) $\log_5\left(\frac{15x}{2x+1}\right)$

$\log_5(15x) - \log_5(2x+1)$

$\boxed{\log_5(15) + \log_5(x) - \log_5(2x+1)}$

$\log_5(5\cdot 3) + \log_5(x) - \log_5(2x+1)$

$\log_5(5) + \log_5(3) + \log_5(x) - \log_5(2x+1)$

$\boxed{1 + \log_5(3) + \log_5(x) - \log_5(2x+1)}$

3.) $\log_5\left(\frac{(x+1)(x-1)}{(x^2-1)}\right)$

$\log_5((x+1)(x-1)) - \log_5(x^2-1)$

$\boxed{\log_5(x+1) + \log_5(x-1) - \log_5(x^2-1)}$

$\log_5(x+1) + \log_5(x-1) - [\log_5(x-1) + \log_5(x+1)]$

$= \boxed{0}$