Function Sketching

1. Linear function.
   - Transformed from parent function $f(x) = x$.
   - Slope indicates steepness and direction.
   - y-intercept indicates shifting up & down.

2. Quadratic function.
   - Transformed from parent function $f(x) = x^2$.
   - Vertex from shifts graph to the new vertex.
   - Negative in front of $x^2$ indicates downward opening.

3. Higher order $x^n$.
   - Higher order makes the graphs "steeper".

4. $f(x) = \frac{1}{x}$
   - $f(x) = \frac{1}{x^2}$
   - $f(x) = -\frac{1}{x^2}$
(5) \( f(x) = \sqrt[x]{x} \), domain is \( \mathbb{R} \)

\[ x^{\frac{1}{2}} \]

(6) \( f(x) = \sqrt[3]{x} = x^{\frac{1}{3}} \), domain is \( \mathbb{R} \)

To find some points on the graph, given the equation, just plug in some values for \( x \) and find the corresponding \( y \) value (\( f(x) \)).

\( f(x) = (x-2)^2 + 7 \)

The vertex of this parabola is \( (2, 7) \)

\[
\begin{array}{c|c|c}
 x & y = f(x) & (x-2)^2 + 7 \\
4 & (4-2)^2 + 7 = 2^2 + 7 = 4 + 7 = 11 & (4, 11) \\
1 & (1-2)^2 + 7 = (-1)^2 + 7 = 1 + 7 = 8 & (1, 8) \\
\end{array}
\]
1. Consider the following function:

\[ y = -\frac{4}{5}x^5 \]

(i) What's the general shape of the function.

From \( f(x) = x^5 \):

\[ \begin{align*}
\text{sign} & \quad \rightarrow \quad \text{negative} \\
\text{flatten} & \quad \rightarrow \quad \frac{\text{by } \frac{4}{5}}{}
\end{align*} \]

(ii) Enter the coordinates to plot points on the graph.

\[
\begin{array}{c|c}
X & y = f(x) = -\frac{4}{5}x^5 \\
\hline
1 & -\frac{4}{5} (1)^5 = -\frac{4}{5} (1) = -\frac{4}{5} \quad (1, -\frac{4}{5}) \\
-1 & -\frac{4}{5}(-1)^5 = -\frac{4}{5}(-1) = \frac{4}{5} \quad (-1, \frac{4}{5}) \\
\end{array}
\]

2. Consider the following function.

\[ p(x) = \frac{5}{2}|x| \]

(i) General shape?

From \( f(x) = |x| \):

\[ \begin{align*}
\text{steepen} & \quad \rightarrow \quad \frac{\text{by } \frac{5}{2}}{}
\end{align*} \]

(ii) Enter some coordinates to plot.

\[
\begin{array}{c|c}
X & y = f(x) = \frac{5}{2}|x| \\
\hline
2 & \frac{5}{2}|2| = \frac{5}{2} \cdot 2 = 5 \\
-2 & \frac{5}{2}|-2| = \frac{5}{2} \cdot 2 = 5 \\
\end{array}
\]
Piecewise Function.
- The function is divided into parts.
- Graph it according to which domain you're in.

Consider the following function:
\[ y(x) = \begin{cases} 
-6x - 18 & \text{if } x \leq -3 \\
\frac{1}{7}x^2 & \text{if } x > -3 
\end{cases} \]

Example:
- \( y(5) = \frac{1}{7}(5)^2 = \frac{25}{7} \) since \( 5 > -3 \).
- \( y(-3) = -6(-3) - 18 = 18 - 18 = 0 \) since \( 0 \leq -3 = -3 \) and \( -3 \leq -3 \).

1. \(-6x - 18\) is a linear graph going downward.

2. \( \frac{1}{7}x^2 \) is quadratic, vertex at \((0,0)\), flatter.

3. Put it together.
Consider the following piecewise function
\[ f(x) = \begin{cases} 
\frac{5}{3}x - 3 & \text{if } x < -4 \\
3x + 4 & \text{if } x \geq -4 
\end{cases} \]
\[ f(-4) = 3(-4) + 4 = -12 + 4 = -8 \quad \text{since } -4 < -4. \]
\[ f(-8) = -8 - 3 = -11, \quad \text{since } -8 < -4. \]
\[ f(-1) = 3(-1) + 4 = -3 + 4 = 1 \quad \text{since } -1 > -4. \]

Consider the following piecewise-defined function.
\[ f(x) = \begin{cases} 
x^2 - x - 7 & \text{if } x \leq 2 \\
4x - 5 & \text{if } x > 6. 
\end{cases} \]
\[ f(4) \text{ is undefined because } 4 \not\leq 2 \text{ and } 4 \not> 6. \]
\[ f(10) = 4(10) - 5 = 40 - 5 = 35 \quad \text{since } 10 > 6. \]
\[ f(2) = 2^2 - 2 - 7 = 4 - 2 - 7 = 2 - 7 = -5 \quad \text{since } 2 \leq 2. \]

Consider the following function
\[ k(x) = \begin{cases} 
\frac{-5x - 10}{3} & \text{if } x \leq -2 \\
\frac{3}{5}x^2 & \text{if } x > -2. 
\end{cases} \]

1. \((2, -2)\)

2. \((2, \infty)\)
Basic Functions

- $f(x) = x$
- $f(x) = x^3$
- $f(x) = x^4$
- $f(x) = \frac{1}{x}$
- $f(x) = \frac{1}{x^2}$
\( f(x) = x^{\frac{1}{2}} \) or \( f(x) = \sqrt{x} \)

\( f(x) = x^{\frac{1}{3}} \) or \( f(x) = \sqrt[3]{x} \)

\( f(x) = \lfloor x \rfloor \) or \( f(x) = \lceil x \rceil \)
Horizontal Shift

\[ g(x) = (x+y)^2 \]

\[ f(x) = x^2 \]

Vertical Shift

\[ f(x) = x^2 \]

\[ g(x) = x^2 - 3 \]

Vertical + Horizontal Shift

Reflecting over the x-axis

\[ f(x) = x \]

\[ g(x) = -\sqrt{x} \]

Reflecting over the y-axis

\[ g(x) = \sqrt{-x} \]
If a function has been transformed from a simpler function, evaluate the transformations in this order:

1) Horizontal Shift
2) Stretching or Compressing
3) Reflections
4) Vertical Shift

\[ f(x) = x^2 \]
\[ g(x) = 5x^2 \]

\[ f(x) = x^2 \]
\[ g(x) = 2x^2 \]

Does \( f(-x) = f(x) \) for every \( x \) in the domain?
If "Yes" this is an **Even Function**.

Ex. \( f(x) = x^2 \)
\[ f(-3) = (-3)^2 = 9 \]
\[ f(3) = 3^2 = 9 \]
\[ f(-3) = f(3) \]

Does \( f(-x) = -f(x) \) for every \( x \) in the domain?
If "Yes" this is an **Odd Function**.

Ex. \( f(x) = x^3 \)
\[ f(-2) = (-2)^3 = -8 \]
\[ f(2) = (2)^3 = 8 \]
\[ f(-2) = -f(2) \]