4.2a Linear and Quadratic Functions

Form of expression

(1) Linear Function
\[ f(x) = mx + b \]

(2) Quadratic Function.

1. Vertex form:
\[ f(x) = (x - h)^2 + k \]
This gives you the vertex of the parabola, which is \((h, k)\).

E.g., \( f(x) = (x - 3)^2 + 7 \), vertex \((3, 7)\)
\( f(x) = (x + 5)^2 - 3 \), vertex \((-5, -3)\)
\( f(x) = x^2 + 6 = (x + 0)^2 + 6 \), vertex \((0, 6)\)
\( f(x) = (x - 5)^2 + (x - 5)^2 + 0 \), vertex \((5, 0)\).

2. Standard form:
\[ f(x) = ax^2 + bx + c \]

The vertex of this function is at \(x = -\frac{b}{2a}\)
and the corresponding \(y\)-value is \(f(x)\) evaluated at \(-\frac{b}{2a}\)
1. Consider the following function:

\[ a(x) = (4x - 18) - (-14 + 6x) \]

\[ a(x) = 4x - 18 + 14 - 6x \]

\[ = (-2x - 4) \]

Slope is \( m = -2 \).

y-intercept is \((0, -4)\).

2. Find two points on the line to graph the function:

\[ a(x) = -2x - 4 \]

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>-2(2) - 4 = -8</td>
</tr>
<tr>
<td>4</td>
<td>-2(4) - 4 = -12</td>
</tr>
</tbody>
</table>

So the two points to plot can be \((2, -8), (4, -12)\).
12. Consider the following function.

\[ h(x) = (x-2)^2 - 9. \]

(1) Find the vertex

(2, -9)

(2) Find the x-intercepts, if any.

Let \( 0 = (x-2)^2 - 9 \)

\[ 9 = (x-2)^2 \]

\[ (x-2)^2 = 9 \]

\[ \sqrt{(x-2)^2} = \sqrt{9} \]

\[ |x-2| = 3 \]

\[ x-2 = \pm 3 \]

Either \( x-2 = 3 \) or \( x-2 = -3 \)

\[ x = 5 \] or \[ x = -1 \]

So x-intercepts are (5,0) and (-1,0).

(3) Find two points on the parabola other than the vertex and x-intercepts.

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
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<tbody>
<tr>
<td>3</td>
<td>(3-2)^2 - 9 = 1 - 9 = -8</td>
</tr>
<tr>
<td>6</td>
<td>(6-2)^2 - 9 = 4^2 - 9 = 16 - 9 = 7</td>
</tr>
</tbody>
</table>

Two points are (3, -8) and (6, 7).
(3) Consider the following function.

\[ f(x) = x^2 + 5x + 6. \]

1. Find the vertex:

\[ x = -\frac{b}{2a} = -\frac{5}{2(1)} = -\frac{5}{2} \]

\[ f\left(-\frac{5}{2}\right) = \left(-\frac{5}{2}\right)^2 + 5\left(-\frac{5}{2}\right) + 6 \]

\[ = \frac{25}{4} - \frac{25}{2} + 6 \]

\[ = -\frac{25}{4} + 6 \]

\[ = -\frac{25}{4} + \frac{24}{4} \]

\[ = -\frac{1}{4} \]

2. Find the x-intercepts, if any.

\[ x^2 + 5x + 6 = 0 \]

\[ (x+3)(x+2) = 0 \]

\[ (x+3) = 0 \quad \text{or} \quad (x+2) = 0. \]

\[ x = -3 \quad \text{or} \quad x = -2. \]

So the x-intercepts are \((-3,0)\) and \((-2,0)\).

3. Find two additional points,

\[ f(1) = 1^2 + 5(1) + 6 = 1 + 5 + 6 = 12 \quad (1,12) \]

\[ f(-1) = (-1)^2 + 5(-1) + 6 = 1 - 5 + 6 = 2 \quad (-1,2) \]

\[ f(0) = 0^2 + 5(0) + 6 = 6 \quad (0,6) \]