Equations Involving Absolute Value

- Absolute Value is NEVER negative
  
  Definition of the absolute value of a number is the distance of that number to zero on the number line.

0 Steps to solve an equation with absolute values

1) Isolate the absolute value expression (not necessarily the variable)

2) Rewrite expression as two equations without absolute values

Examples:

1) \( |x| = 5 \)

\[ \Rightarrow x = 5 \text{ or } x = -5 \]

Solution Set: \{5, -5\}

2) \( |2x + 3| = 7 \)

\[ \Rightarrow 2x + 3 = 7 \text{ or } 2x + 3 = -7 \]

\[ 2x = 4 \quad 2x = -10 \]

\[ x = 2 \quad x = -5 \]

Solution Set: \{2, -5\}

3) \( |x - 4| = |2x + 1| \)

\[ \Rightarrow x - 4 = 2x + 1 \quad \text{or} \quad x - 4 = -(2x + 1) \]

\[ -4 = x + 1 \quad \text{or} \quad x - 4 = -2x - 1 \]

\[ -5 = x \quad 3x = -1 \]

Solution Set: \{-5\}
(4) \[ |6x - 7| + 5 = 3 \]
\[ |6x - 7| = -2 \]

Since the absolute value can never be negative, the equation has NO SOLUTION. Therefore the solution set is \( \emptyset \).

2.16 Word Problems

Hawkes Practice:

1. Solve the following formula for the indicated variable:
   \[ C = 2\pi r \] solve for \( r \).
   \[ r = \frac{C}{2\pi} \]

2. Two trucks leave a warehouse at the same time. One travels due north at an average speed of 48 miles per hour, and the other travels due south at an average speed of 46 miles per hour. After how many hours will the two trucks be 470 miles apart?

   Notice after 1 hour, the trucks would be
   \[ (48 + 46) \frac{\text{miles}}{\text{hour}} \cdot (1 \text{ hr}) = 94 \text{ miles, apart} \]

   So it takes \( \frac{470 \text{ miles}}{94 \text{ miles/hour}} = 5 \text{ hours for them to be 470 miles apart} \).

   Alternatively, set up a proportion,
   \[ \frac{\text{dist}_1}{\text{time}_1} = \frac{\text{dist}_2}{\text{time}_2} \]
   \[ \frac{94}{5} = \frac{470}{x} \]
   \[ 94x = 470 \cdot 5 \]
   \[ x = 5 \]
(3) Solve the following formula for the indicated variable.

\[ v^2 = v_0^2 + 2ax \] solve for \( a \).

\[ v^2 - v_0^2 = 2ax \]

\[ \frac{v^2 - v_0^2}{2x} = a \]

Quiz tomorrow (Friday 1/24).