04-09-2019

HW Help Session

■ $\sum_{n=0}^{\infty} (3n+1)!\,(2x-10)^n$

$$L = \lim_{n\to\infty} \left| \frac{(3n+4)!\,(2x-10)^{n+1}}{(3n+1)!\,(2x-10)^n} \right| = \lim_{n\to\infty} |2x-10|(3n+4)(3n+3)(3n+2) \longrightarrow \infty$$

$(3n+4)! = (3n+4)(3n+3)(3n+2)\cancel{(3n+1)!}$

provided $x \neq 5$.

Remember every power series always converges for $a=5$.

Then our interval is $\{5\}$, the radius is zero.

■ Taylor Series: $h(x) = 12(1-3x)^{-6}$ about $x=1$

$n=0 \quad 12(1-3x)^{-6}$

$n=1 \quad 12(+6)(1-3x)^{-7}(+3)$

$n=2 \quad 12(6)(+7)(1-3x)^{-8}3^2$

$n=3 \quad 12(6)(7)(8)(1-3x)^{-9}3^3$

$n=4 \quad 12(6)(7)(8)(9)(1-3x)^{-10}3^4$

Thus: $f^{(n)}(x) = \cancel{12}\cdot 3^n (1-3x)^{-(6+n)} \dfrac{(n+5)!}{5!}$, $1\cdot 2\cdot \cancel{3}\cdot \cancel{4}\cdot 5$

$$= \frac{3^n}{10}(1-3x)^{-(6+n)}(n+5)!$$

$$f^{(n)}(1) = \frac{3^n}{10}\underbrace{(1-3)}_{-2}^{-(6+n)}(n+5)!$$

$$= \frac{3^n}{10}(-1)^{-(6+n)}2^{-(6+n)}(n+5)!$$

$$= (-1)^{-(6+n)}\frac{3^n}{10\cdot 2^{+6}\cdot 2^n}(n+5)!$$

$$= (-1)^{-(6+n)}\left(\frac{3}{2}\right)^n (n+5)!\,\frac{1}{640}$$

Thus our Taylor series will be:

$$h(x) = \sum_{n=0}^{\infty} (-1)^{-(6+n)}\left(\frac{3}{2}\right)^n \frac{(n+5)!}{n!}\frac{1}{640}(x-1)^n$$

■ $f(x) = \dfrac{x^2}{4+\sqrt[3]{x}}$ [Power Series representation].

$$f(x) = \frac{x^2}{4\left(1+\frac{x^{1/3}}{4}\right)} = \frac{x^2}{4\left(1-\left(-\frac{x^{1/3}}{4}\right)\right)} = \frac{x^2}{4}\cdot\frac{1}{1-\underbrace{\left(-\frac{x^{1/3}}{4}\right)}_{p}} \longrightarrow \sum p^n$$

Thus:

$$f(x) = \frac{x^2}{4}\sum_{n=0}^{\infty}\left(-\frac{x^{1/3}}{4}\right)^n = \frac{x^2}{4}\sum_{n=0}^{\infty}(-1)^n\frac{x^{n/3}}{4^n} = \underbrace{\sum_{n=0}^{\infty}(-1)^n\frac{x^{\frac{n}{3}+2}}{4^{n+1}}}_{\text{series represent.}}$$

Provided:

$$\left|-\frac{x^{1/3}}{4}\right| < 1$$

$$|x^{1/3}| < 4$$

$$|x| < 64$$

■ $\sum_{n=0}^{\infty} \dfrac{5^{1-n}(2x-4)^n}{n^2+1}$

$$L = \lim_{n\to\infty}\left|\frac{5^{\cancel{-n}}(2x-4)^{n+1}}{(n+1)^2+1}\cdot\frac{n^2+1}{5^{\cancel{1-n}}\cancel{(2x-4)^n}}\right| = \lim_{n\to\infty}\left|\frac{(n^2+1)(2x-4)}{((n+1)^2+1)\cdot 5}\right| = \frac{|2x-4|}{5}\lim_{n\to\infty}\left|\frac{n^2+1}{(n+1)^2+1}\right|$$

$$= \frac{|2x-4|}{5} < 1$$

$$|2x-4| < 5$$

$$2|x-2| < 5$$

$$|x-2| < \frac{5}{2}$$

$$-\frac{1}{2} = -\frac{5}{2}+2 < x < \frac{5}{2}+2 = \frac{9}{2} \longrightarrow -\frac{1}{2} < x < \frac{9}{2}\text{ , Radius: } 5/2.$$

Checking endpoints:

$x=-1/2$: $\displaystyle\sum_{n=0}^{\infty} \frac{5^{1-n}}{n^2+1}(-5)^n$

$= \displaystyle\sum_{n=0}^{\infty} \frac{5^{1-\cancel{n}}(-1)^n \cancel{5^n}}{n^2+1}$

$= 5\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{n^2+1}$ converges by Alt. Ser. Test.

$x=9/2$: $\displaystyle\sum_{n=0}^{\infty} \frac{5^{1-n}}{n^2+1} 5^n = \sum_{n=0}^{\infty} \frac{5}{n^2+1}$

$= 5\displaystyle\sum_{n=0}^{\infty} \frac{1}{n^2+1} \le 5\sum_{n=0}^{\infty} \frac{1}{n^2}$

convergent p-series

Thus, interval of convergence: $-\frac{1}{2} \le x \le \frac{9}{2}$.