4-18-2019

Equations of planes:

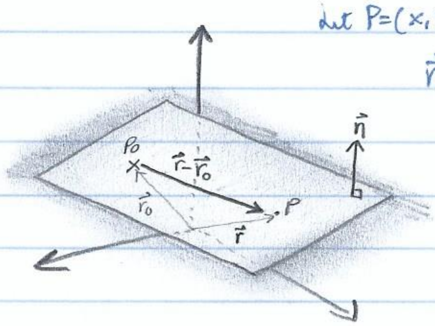
What do I need? A point on the plane: $P_0 = (x_0, y_0, z_0)$, $\vec{r}_0 = \langle x_0, y_0, z_0 \rangle$

A vector normal to the plane (orthogonal). $\vec{n} = \langle a, b, c \rangle$

Let $P = (x, y, z)$ be any other pt on the plane ($\vec{r} = \langle x, y, z \rangle$).

vector form of plane equation.

$\vec{n}$ and $\vec{r} - \vec{r}_0$ are orthogonal vectors, i.e. $\vec{n} \cdot (\vec{r} - \vec{r}_0) = 0$



$$\langle x - x_0, y - y_0, z - z_0 \rangle \cdot \langle a, b, c \rangle = 0$$

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

Equation of the plane

$$ax + by + cz = d = ax_0 + by_0 + cz_0$$

Example: Find the equation of the plane containing:

P_0 ← $P = (1, -2, 0)$, $Q = (3, 1, 4)$, $R = (0, -1, 2)$.

→ Need $\vec{n}$ (normal to plane):

$\vec{PQ} = \langle 2, 3, 4 \rangle$ $\quad \vec{PQ} \times \vec{PR} = \begin{vmatrix} i & j & k \\ 2 & 3 & 4 \\ -1 & 1 & 2 \end{vmatrix} = i \begin{vmatrix} 3 & 4 \\ 1 & 2 \end{vmatrix} - j \begin{vmatrix} 2 & 4 \\ -1 & 2 \end{vmatrix} + k \begin{vmatrix} 2 & 3 \\ -1 & 1 \end{vmatrix}$

$\vec{PR} = \langle -1, 1, 2 \rangle$

$$= \hat{i}(2) - \hat{j}(8) + \hat{k}(5) = \overset{a}{2}\hat{i} - \overset{b}{8}\hat{j} + \overset{c}{5}\hat{k}$$

→ Equation:

$$2(x-1) - 8(y+2) + 5z = 0$$

$$2x - 8y + 5z = 18$$

Example: Determine if $\underset{\text{plane}}{-x + 2z = 10}$ and $\underset{\text{line}}{\vec{r} = \langle 5, 2-t, 10+4t \rangle}$ are parallel, orthogonal or neither.

$\vec{v} = \langle 0, -1, 4 \rangle$ is parallel to $\vec{r}$.

$\vec{n} = \langle -1, 0, 2 \rangle$ is normal to the $-x + 2z = 10$ plane.

→ If $\vec{v}$ and $\vec{n}$ are parallel, then $\vec{r}$ and the plane are orthogonal.

Notice $\vec{v}$ and $\vec{n}$ are not scalar multiples of each other, so not parallel either.

Thus, the line and the plane are not orthogonal.

→ If $\vec{v}$ and $\vec{n}$ are orthogonal, then the line and plane are parallel.

Let's check: $\vec{v} \cdot \vec{n} = \langle 0, -1, 4 \rangle \cdot \langle -1, 0, 2 \rangle = 0(-1) + (-1)0 + 8 = 8 \neq 0$.

Not orthogonal ($\vec{v}$ and $\vec{n}$).

Then plane and line are not parallel.

ANSWER: Neither.