Equations of planes:

What do I need? A point on the plane: \( \mathbf{p} = (x_0, y_0, z_0) \), \( \mathbf{p}_0 = (x_0, y_0, z_0) \)

A vector normal to the plane (orthogonal). \( \mathbf{n} = \langle a, b, c \rangle \)

Let \( \mathbf{p} = (x, y, z) \) be any other pt on the plane \( (\mathbf{p} = (x, y, z)) \).

\( \mathbf{n} \) and \( \mathbf{r} - \mathbf{r}_0 \) are orthogonal vectors, i.e. \( \mathbf{n} \cdot (\mathbf{r} - \mathbf{r}_0) = 0 \)

\[ \langle x-x_0, y-y_0, z-z_0 \rangle \cdot \langle a, b, c \rangle = 0 \]

\[ a(x-x_0) + b(y-y_0) + c(z-z_0) = 0 \]

Equation of the plane

\[ ax + by + cz = d = ax_0 + by_0 + cz_0 \]

Example: Find the equation of the plane containing:

\( \mathbf{p}_1 = (1, 2, 0) \), \( \mathbf{p}_2 = (3, 1, 4) \), \( \mathbf{p}_3 = (0, -1, 2) \).

\[ \vec{\mathbf{n}} \text{ (normal to plane):} \]

\[ \mathbf{p}_0 \times \mathbf{p}_0 = \begin{vmatrix} 1 & 3 & 4 \\ 2 & 3 & 4 \\ -1 & 1 & 2 \end{vmatrix} = 6 \]

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\[ \mathbf{n} = \langle 2, 3, 4 \rangle \]

Equation:

\[ 2(x-1) + 3(y+2) + 4z = 0 \]

\[ 2x - 8y + 4z = 18 \]

Example: Determine if \(-x + 2z = 10\) and \( \mathbf{r} = \langle 5, 2, -10 + 4t \rangle \) are parallel, orthogonal or neither.

\( \mathbf{v} = \langle 0, -1, 4 \rangle \) is parallel to \( \mathbf{r} \).

\( \mathbf{n} = \langle -1, 0, 2 \rangle \) is normal to the \(-x + 2z = 10\) plane.

If \( \mathbf{v} \) and \( \mathbf{n} \) are parallel, then \( \mathbf{r} \) and the plane are orthogonal.

Notice \( \mathbf{v} \) and \( \mathbf{n} \) are not scalar multiples of each other, so not parallel either.

Thus, the line and the plane are not orthogonal.

If \( \mathbf{v} \) and \( \mathbf{n} \) are orthogonal, then the line and plane are parallel.

Let's check: \( \mathbf{v} \cdot \mathbf{n} = 0 \) \( \langle 0, -1, 4 \rangle \cdot \langle -1, 0, 2 \rangle = 0(-1) + 0 + 8 = 8 \neq 0 \).

Not orthogonal (\( \mathbf{v} \) and \( \mathbf{n} \)).

Thus, plane and line are not parallel.

Answer: Neither.