04-17-2019

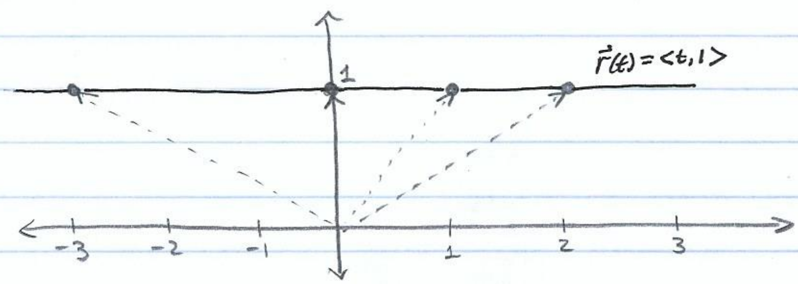
Equation of a line:

Vector Functions: $f: \mathbb{R} \to \mathbb{R}^n$

ex: $\vec{r}(t) = \underbrace{\langle t, 1 \rangle}_{\text{position vectors}}$ where $t \in \mathbb{R}$.

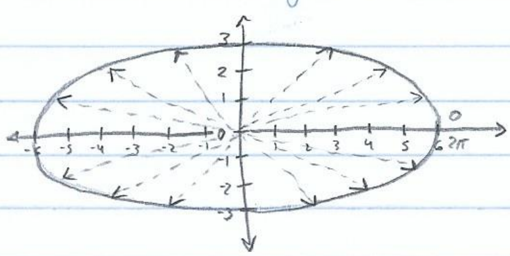
How do we plot this?

$\vec{r}(2) = \langle 2, 1 \rangle$; $\vec{r}(1) = \langle 1, 1 \rangle$; $\vec{r}(0) = \langle 0, 1 \rangle$; $\vec{r}(-3) = \langle -3, 1 \rangle$



What about? $\vec{r}(t) = \langle 6\cos t, 3\sin t \rangle$ } vector form

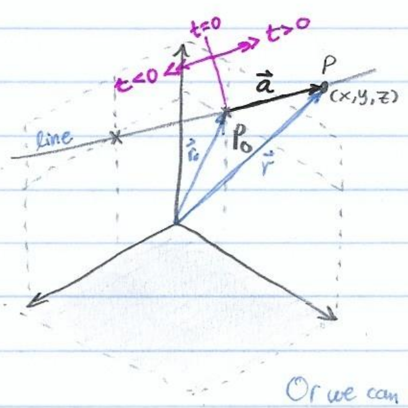
$x = 6\cos t$
$y = 3\sin t$ } parametric form



For a line we are going to need:

- A point on the line $(x_0, y_0, z_0) = P_0$
- A vector that points in the same direction of line $\vec{v} = \langle a, b, c \rangle$ (parallel)

$\hookrightarrow \vec{r}_0$ → Let's call $P = \langle x, y, z \rangle$ any pt on the line $\hookrightarrow \vec{r} = \langle x, y, z \rangle$



→ $\vec{a}$ is on the line, so it's parallel to $\vec{v}$.

So $\exists t \in \mathbb{R}$ s.t. $\vec{a} = t\vec{v}$.

→ Thus, from our image we can see that:

$\underset{\vec{r}(t)}{\vec{r}} = \vec{r}_0 + t\vec{v}$ $\quad \boxed{\vec{r}(t) = \vec{r}_0 + t\vec{v}}$ Vector Form

Or we can write it as: $\vec{r}(t) = \langle x_0, y_0, z_0 \rangle + t\langle a, b, c \rangle$

$= \langle x_0 + ta, y_0 + tb, z_0 + tc \rangle$

Parametric Form
$$\boxed{\begin{aligned} x &= x_0 + at \\ y &= y_0 + bt \\ z &= z_0 + ct \end{aligned}}$$

Ex: Find the equation of the line through: $(1, -3, 4)$ and $(3, -7, 2)$

- Point on the line: $p = (1, -3, 4)$
- Parallel vector: $\vec{v} = \vec{PQ} = \vec{Q} - \vec{P} = \langle 2, -4, -2 \rangle$

Then: our equation of the line is:

$$\vec{r}(t) = \langle 1, -3, 4 \rangle + t \langle 2, -4, -2 \rangle$$

or: $\vec{r}(t) = \langle 1 + 2t, -3 - 4t, 4 - 2t \rangle$ ...... parametric .... $x = 1 + 2t$, $y = -3 - 4t$, $z = 4 - 2t$

Ex: Find eqt'n of line through $(0, -6, 2)$ and parallel to $x = 10, y = -4 + t, z = 12t$ (*)

- point on the line: $(0, -6, 2)$
- parallel vector: $\vec{v} = (0, 1, 12)$

→ The coefficients of t (param. form) give us a vector parallel to the eqt'n (*) of the line, thus also parallel to the line we want to find

Then: $\vec{r}(t) = (0, -6, 2) + t(0, 1, 12)$

Parametric form:
$x = 0$
$y = -6 + t$
$z = 2 + 12t$