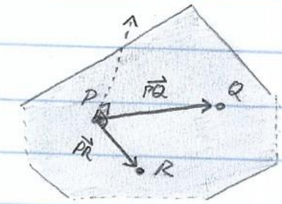
04-15-2019

**Ex:** A plane has the 3 points $P=(1,0,0)$, $Q=(1,1,1)$, $R=(2,-1,3)$. Let's find a vector that is orthogonal to the plane.

→ What to do? cross products!

$\vec{PQ} = \langle 0,1,1 \rangle \quad (Q-P)$

$\vec{PR} = \langle 1,-1,3 \rangle \quad (R-P)$




→ Orthogonal vector:

$$\vec{PQ} \times \vec{PR} = \begin{vmatrix} i & j & k \\ 0 & 1 & 1 \\ 1 & -1 & 3 \end{vmatrix} = \hat{i}\begin{vmatrix} 1 & 1 \\ -1 & 3 \end{vmatrix} - \hat{j}\begin{vmatrix} 0 & 1 \\ 1 & 3 \end{vmatrix} + \hat{k}\begin{vmatrix} 0 & 1 \\ 1 & -1 \end{vmatrix}$$

$$= i(3+1) - j(0-1) + k(-1) = 4\hat{i} + \hat{j} - \hat{k}$$

- Given 3 vectors $\vec{a}, \vec{b}, \vec{c}$, we can use these to form a parallelopiped. The volume of this object is given by $V = |\vec{a} \cdot (\vec{b} \times \vec{c})|$.

→ Notice that if this volume is zero, then we have the 3 vectors on the same plane! They're coplanar vectors.

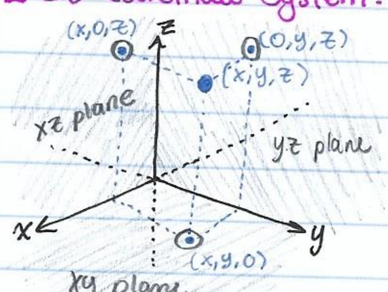
**Ex:** Are $\underset{\vec{a}}{\langle 1,4,-7 \rangle}, \underset{\vec{b}}{\langle 2,-1,4 \rangle}, \underset{\vec{c}}{\langle 0,-9,18 \rangle}$ on the same plane?

- $$\vec{a} \cdot (\vec{b} \times \vec{c}) = \begin{vmatrix} 1 & 4 & -7 \\ 2 & -1 & 4 \\ 0 & -9 & 18 \end{vmatrix} = 1\begin{vmatrix} -1 & 4 \\ -9 & 18 \end{vmatrix} - 4\begin{vmatrix} 2 & 4 \\ 0 & 18 \end{vmatrix} - 7\begin{vmatrix} 2 & -1 \\ 0 & -9 \end{vmatrix}$$

$$= 1(-18+36) - 4(36) - 7(-18)$$

$$= 18 - 8(18) + 7(18) = 8(18) - 8(18) = 0$$

## 3D Coordinate System:




→ All pts are given by triples $(x,y,z)$, and have projections onto the $xy$, $xz$, and $yz$ planes.

→ Distance between two points:

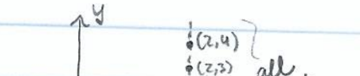
$$d(P,Q) = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$$

→ Remember:

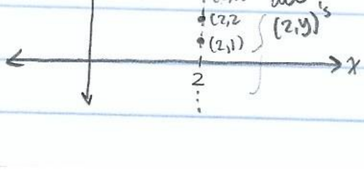
$$\underset{\text{Circle}}{x^2+y^2=r^2} \qquad \underset{\text{Sphere}}{x^2+y^2+z^2=r^2}$$

**Ex:** "Sketch" $x=2$.

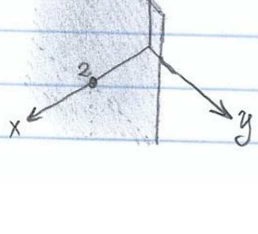
1D ($\mathbb{R}$)




2D ($\mathbb{R}^2$)

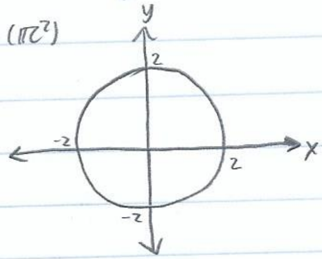



3D ($\mathbb{R}^3$)



Ex "Sketch" $x^2+y^2=4$.

2D ($\mathbb{R}^2$)




3D ($\mathbb{R}^3$)