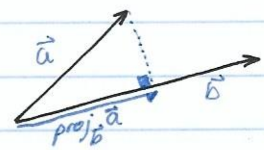
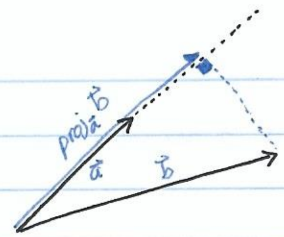
4/12/19

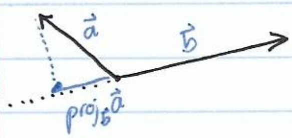
■ Projections:





Projections are the shadow of a vector on the line defined by another vector.

How to get this projection:



$$proj_{\vec{b}}\vec{a} = \frac{\vec{a}\cdot\vec{b}}{\|\vec{b}\|^2}\vec{b} \quad \text{or} \quad proj_{\vec{a}}\vec{b} = \frac{\vec{a}\cdot\vec{b}}{\|\vec{a}\|^2}\vec{a}$$

Ex:

Find projection of $\vec{u}=\langle 2,1,-1\rangle$ onto $\vec{v}=\langle 1,0,-2\rangle$.

$$proj_{\vec{v}}\vec{u} = \frac{\vec{u}\cdot\vec{v}}{\|\vec{v}\|^2}\vec{v} = \frac{4}{5}\langle 1,0,-2\rangle = \langle \tfrac{4}{5}, 0, \tfrac{-8}{5}\rangle$$

Ex of $\vec{u}=\langle 2,1,-1\rangle$ onto $\vec{v}=\langle 1,0,2\rangle$.

Note $\vec{u}\cdot\vec{v}=0$ Thus, the vectors are orthogonal, so the projection is the zero vector, or it just doesn't exist.

• The cross product: [requires 3D vectors]

Given the two vectors $\vec{a}=\langle a_1,a_2,a_3\rangle$, $\vec{b}=\langle b_1,b_2,b_3\rangle$

$$\vec{a}\times\vec{b} = \langle a_2b_3 - a_3b_2,\ a_3b_1 - a_1b_3,\ a_1b_2 - a_2b_1\rangle$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \hat{i}\begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} - \hat{j}\begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} + \hat{k}\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$$

Ex: Given $\vec{a}=\langle 2,1,-1\rangle$, $\vec{b}=-3\hat{i}+4\hat{j}+\hat{k}=\langle -3,4,1\rangle$

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ -3 & 4 & 1 \end{vmatrix} = \hat{i}\begin{vmatrix} 1 & -1 \\ 4 & 1 \end{vmatrix} - \hat{j}\begin{vmatrix} 2 & -1 \\ -3 & 1 \end{vmatrix} + \hat{k}\begin{vmatrix} 2 & 1 \\ -3 & 4 \end{vmatrix}$$

$$= \hat{i}(1+4) - \hat{j}(2-3) + \hat{k}(8+3)$$

$$= 5\hat{i} + \hat{j} + 11\hat{k}$$

• Facts/Properties:

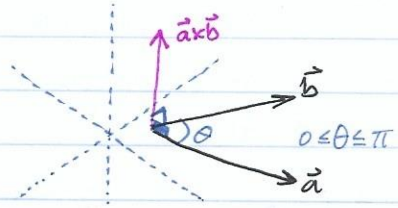
$$\vec{a}\times\vec{b} = -(\vec{b}\times\vec{a})$$

$$\vec{b}\times\vec{a} = \begin{vmatrix} i & j & k \\ -3 & 4 & 1 \\ 2 & 1 & -1 \end{vmatrix} = \hat{i}\begin{vmatrix} 4 & 1 \\ 1 & -1 \end{vmatrix} - \hat{j}\begin{vmatrix} -3 & 1 \\ 2 & -1 \end{vmatrix} + \hat{k}\begin{vmatrix} -3 & 4 \\ 2 & 1 \end{vmatrix}$$

$$= \hat{i}(-4-1) - j(3-2) + \hat{k}(-3-8)$$

$$= -5\hat{i} - \hat{j} - 11\hat{k}$$

- Facts/properties:
  - $\vec{a}\times\vec{b}$ is perpendicular to both $\vec{a}$ and $\vec{b}$.



- $\|\vec{a}\times\vec{b}\| = \|\vec{a}\|\|\vec{b}\| \sin\theta$
- If $\vec{a} \parallel \vec{b}$ (both nonzero), i.e. $\theta = 0, \pi$, $\sin\theta = 0$

  Then $\|\vec{a}\times\vec{b}\| = 0 = \|\vec{a}\|\|\vec{b}\|\sin(0,\pi) = \|a\|\|b\| 0$

  and also $\vec{a}\times\vec{b} = \vec{0}$
- Since $\vec{a}\times\vec{b}$ is orthogonal to both $\vec{a}$ and $\vec{b}$:

  $(\vec{a}\times\vec{b})\cdot\vec{a} = 0$ and $(\vec{a}\times\vec{b})\cdot\vec{b} = 0$
- $\vec{u}\times(\vec{v}+\vec{w}) = (\vec{u}\times\vec{v}) + (\vec{u}\times\vec{w})$
- $\vec{u}\cdot(\vec{v}\times\vec{w}) = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$