04-11-2019

Ex: Determine if the vectors are parallel:

(a) $\vec{a} = \langle 2, -4, 1 \rangle$, $\vec{b} = \langle -6, 12, -3 \rangle$

Notice $\vec{b} = -3\vec{a}$. Thus, they're parallel.

(b) $\vec{a} = \langle 1, 4 \rangle$, $\vec{b} = \langle -3, 7 \rangle$

Not scalar multiples of each other.

(c) $\vec{a} = \langle 1, 0 \rangle$, $\vec{b} = \langle 4, 3 \rangle$

Not parallel.

(d) $\vec{a} = \langle 0, 0 \rangle$, $\vec{b} = \langle 3, -7 \rangle$

Parallel vectors need to be nonzero.

Ex: Unitize the following vectors:

(a) $\vec{a} = \langle 3, -1, 4 \rangle$

$\|\vec{a}\| = \sqrt{(3)^2 + (-1)^2 + (4)^2} = \sqrt{26}$

then $\hat{a} = \frac{1}{\sqrt{26}} \langle 3, -1, 4 \rangle$ is of length 1. -----> You can check!

FACT: For a given nonzero vector $\vec{a}$, the vector $\hat{a} = \frac{\vec{a}}{\|\vec{a}\|}$ is a unit vector.

Notice the following: For any vector $\vec{a} = \langle a_1, a_2, a_3 \rangle$

$$\langle a_1, a_2, a_3 \rangle = a_1 \langle 1, 0, 0 \rangle + a_2 \langle 0, 1, 0 \rangle + a_3 \langle 0, 0, 1 \rangle$$
$$= a_1 \hat{i} + a_2 \hat{j} + a_3 \hat{k}.$$

THE DOT PRODUCT:

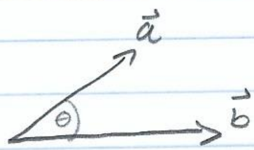
Given $\vec{a} = \langle a_1, a_2, a_3 \rangle$ and $\vec{b} = \langle b_1, b_2, b_3 \rangle$

The dot product is computed as follows: $\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$

PROPERTIES:

- $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$
- $(\alpha \vec{a}) \cdot \vec{b} = \alpha (\vec{a} \cdot \vec{b}) = \vec{a} \cdot (\alpha \vec{b})$
- $\vec{a} \cdot \vec{0} = 0$
- $\|\vec{a}\| = \sqrt{\vec{a} \cdot \vec{a}}$ or $\|\vec{a}\|^2 = \vec{a} \cdot \vec{a}$
- $\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$
- If $\vec{a} \cdot \vec{a} = 0$ then $\vec{a} = \vec{0}$
- $\vec{a} \cdot \vec{a} \geq 0$ for all $\vec{a}$.

GEOMETRIC INTERPRETATION:



$0 \leq \theta \leq \pi$

0: parallel same direction; π: parallel oppos. direction

$$\vec{a} \cdot \vec{b} = \|\vec{a}\| \|\vec{b}\| \cos\theta \rightarrow \underbrace{\cos\theta = \frac{\vec{a} \cdot \vec{b}}{\|\vec{a}\| \|\vec{b}\|}}_{\text{Useful if we want to find the angle between } \vec{a} \text{ and } \vec{b}.}$$

Ex:

(a) $\vec{v} = \langle 3, -4, -1 \rangle$, $\vec{w} = \langle 0, 5, 2 \rangle$

$\vec{v} \cdot \vec{w} = (3)(0) + (-4)(5) + (-1)(2) = -22$

$\|\vec{v}\| = \sqrt{9+16+1} = \sqrt{26}$ $\quad \|\vec{w}\| = \sqrt{0+25+4} = \sqrt{29}$

$\cos\theta = \dfrac{-22}{\sqrt{26} \cdot \sqrt{29}} = -0.8011927 \Longrightarrow \theta = \cos^{-1}(-0.8011927) = 2.5$ (radians).

(b) $\vec{v} = \langle 6, -2, -1 \rangle$, $\vec{w} = 2\hat{i} + 5\hat{j} + 2\hat{k} = \langle 2, 5, 2 \rangle$

$\vec{v} \cdot \vec{w} = (6)(2) + (-2)(5) + (-1)(2) = 0 \dashrightarrow \cos\theta = 0 \dashrightarrow \theta = \pi/2$

FACT:

Two nonzero vectors $\vec{a}$ and $\vec{b}$ are perpendicular (orthogonal) if $\vec{a} \cdot \vec{b} = 0$

If $\vec{a}$ and $\vec{b}$ are parallel, then $\theta = 0$ or $\theta = \pi$, then $\vec{a} \cdot \vec{b} = \|\vec{a}\| \|\vec{b}\|$ or $\vec{a} \cdot \vec{b} = -\|\vec{a}\| \|\vec{b}\|$.