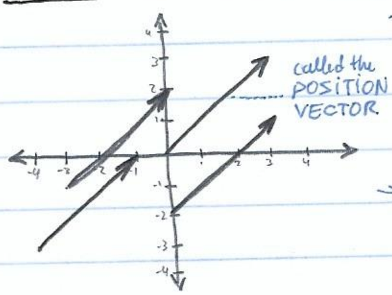
VECTORS:

THE BASICS:



→ All of these vectors represent the vector $\langle 3,3\rangle$
Although the position vector is usually the one to think of when given a vector.

→ If a vector starts at $P=(x_1,y_1,z_1)$ and ends at $Q=(x_2,y_2,z_2)$ then we obtain the vector:

$$\vec{PQ}=(x_2-x_1,\ y_2-y_1,\ z_2-z_1)$$
$$\vec{QP}=(x_1-x_2,\ y_1-y_2,\ z_1-z_2) \quad \text{[The other direction]}$$

→ The magnitude (or length) of the vector $\vec{a}=\langle a_1,a_2,a_3\rangle$ is given by

$$\|\vec{a}\|=\sqrt{a_1^2+a_2^2+a_3^2}$$

Magnitude will never be negative.

• FACT: $\|\vec{a}\|=0$ iff $\vec{a}=\vec{0}=\langle 0,0,\ldots,0\rangle$

Example:

(a) $\vec{b}=\langle 3,-2,1\rangle \quad \|\vec{b}\|=\sqrt{(3)^2+(-2)^2+(1)^2}=\sqrt{9+4+1}=\sqrt{14}$

(b) $\vec{c}=\langle \frac{-2}{\sqrt{5}},\frac{1}{\sqrt{5}}\rangle \quad \|\vec{c}\|=\sqrt{\left(\frac{-2}{\sqrt{5}}\right)^2+\left(\frac{1}{\sqrt{5}}\right)^2}=\sqrt{\frac{4}{5}+\frac{1}{5}}=\sqrt{\frac{5}{5}}=1$

(c) $\vec{i}=\langle 1,0,0\rangle \quad \|\vec{i}\|=\sqrt{1^2+0^2+0^2}=\sqrt{1}=1$

DEF'N: The vector $\vec{u}$ is said to be a unit vector if $\|\vec{u}\|=1$.

STANDARD BASIS VECTORS:

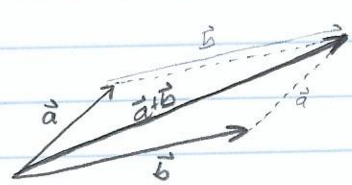
$\vec{i}=\langle 1,0,0\rangle \quad \vec{j}=\langle 0,1,0\rangle \quad \vec{k}=\langle 0,0,1\rangle$

VECTOR ARITMETIC:

$\vec{a}=\langle a_1,a_2,a_3\rangle,\ \vec{b}=\langle b_1,b_2,b_3\rangle$

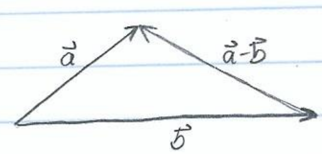
$\vec{a}+\vec{b}=\langle a_1+b_1,\ a_2+b_2,\ a_3+b_3\rangle$ Remember: $\vec{a}+\vec{b}=\vec{b}+\vec{a}$

Geometric Interpretation:



→ Head-to-tail interpretation.
→ Parallelogram interpretation.

$\vec{a}-\vec{b}=\langle a_1-b_1,\ a_2-b_2,\ a_3-b_3\rangle$

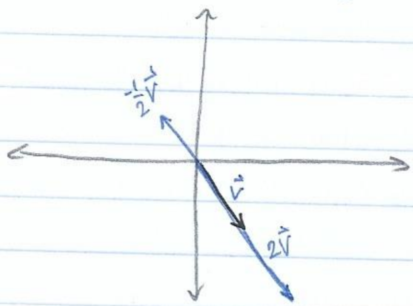


MULTIPLICATION: SCALAR

$$\alpha \vec{a} = \langle \alpha a_1, \alpha a_2, \alpha a_3 \rangle$$

some number

- Geometric interpretation: given $\vec{v} = \langle 1, -2 \rangle$



$2\vec{v} = \langle 2, -4 \rangle$ [stretched]

$-\frac{1}{2}\vec{v} = \langle -\frac{1}{2}, 1 \rangle$ [shrink, oppos. dir.]

FACT: Two vectors, $\vec{a}$ and $\vec{b}$, are parallel if: going in the same direction or opposite direction. Mathematically expressed: they're scalar multiples of the other, there's a number α so that $\vec{a} = \alpha \vec{b}$