03-08-2019

**Ex:** $\displaystyle\sum_{n=1}^{\infty} 9^{-n+2}4^{n+1} = \sum_{n=1} \frac{4^{n+1}}{9^{n-2}} = \sum_{n=1} \frac{4^{n-1}4^2}{9^{n-1}9^{-1}}$

$\displaystyle = \underbrace{144}_{a} \sum_{n=1} \underbrace{\left(\frac{4}{9}\right)}_{r}^{n-1}$ Note: $|r| = \left|\frac{4}{9}\right| < 1$

So this series converges.

$$\frac{a}{1-r} = \frac{144}{1-\frac{4}{9}} = \frac{1296}{5}$$

Now, if we were given $\displaystyle\sum_{n=3}^{\infty} 9^{-n+2}4^{n+1}$, then we know this converges since $\displaystyle\sum_{n=1}(\ldots)$ does.

→ What's the value? $\displaystyle\sum_{n=1}^{\infty} 9^{-n+2}4^{n+1} = 9^1 4^2 + 9^0 4^3 + \sum_{n=3}^{\infty} 9^{-n+2}4^{n+1}$

$$\frac{1296}{5} = 208 + \sum_{n=3}^{\infty} 9^{-n+2}4^{n+1}$$

$$\sum_{n=3}^{\infty} 9^{-n+2}4^{n+1} = \frac{256}{5}$$

**Ex:**

- $\displaystyle\sum_{n=0}^{\infty} \frac{(-4)^{3n}}{5^{n-1}} = \sum_{n=0}^{\infty} \frac{((-4)^3)^n}{5^n 5^{-1}} = \sum_{n=0}^{\infty} \frac{(-64)^n \cdot 5}{5^n} = \sum_{n=0}^{\infty} \underbrace{\left(\frac{-64}{5}\right)}_{r}^{n} \cdot \underset{a}{5}$

$$|r| = \left|\frac{-64}{5}\right| = \frac{64}{5} > 1$$

then this series DIVERGES.

## TELESCOPING SERIES:

**Ex:** Converges? Diverges? If convergent, to what value?

$\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+4n+3}$

- limit of the partial sum:

$\displaystyle S_N = \sum_{i=1}^{N} \frac{1}{i^2+4i+3} = \sum_{i=1}^{N}\left[\frac{1/2}{i+1} - \frac{1/2}{i+3}\right]$ Make sure you have "−" in between and same numerator

$$= \underset{i=1}{\left[\frac{1/2}{2} - \cancel{\frac{1/2}{4}}\right]} + \underset{i=2}{\left[\frac{1/2}{3} - \cancel{\frac{1/2}{5}}\right]} + \underset{i=3}{\left[\cancel{\frac{1/2}{4}} - \cancel{\frac{1/2}{6}}\right]}$$

$$+ \underset{i=4}{\left[\cancel{\frac{1/2}{5}} - \cancel{\frac{1/2}{7}}\right]} + \underset{i=5}{\left[\cancel{\frac{1/2}{6}} - \cancel{\frac{1/2}{8}}\right]} + \ldots$$

$$\left[\cancel{\frac{1/2}{N-3}} - \cancel{\frac{1/2}{N-1}}\right] + \left[\cancel{\frac{1/2}{N-2}} - \cancel{\frac{1/2}{N}}\right] + \left[\cancel{\frac{1/2}{N-1}} - \cancel{\frac{1/2}{N+1}}\right] + \left[\cancel{\frac{1/2}{N}} - \frac{1/2}{N+2}\right] + \left[\cancel{\frac{1/2}{N+1}} - \frac{1/2}{N+3}\right]$$

$$S_N = \frac{1}{2}\left(\frac{1}{2} + \frac{1}{3} - \frac{1}{N+2} - \frac{1}{N+3}\right)$$

$S_N = \frac{1}{2}\left(\frac{5}{6} - \frac{1}{N+2} - \frac{1}{N+3}\right)$ These are called Telescoping Series

$$\lim_{N \to \infty} S_N = \lim_{N \to \infty}\left(\frac{1}{2}\left(\frac{5}{6} - \frac{1}{N+2} - \frac{1}{N+3}\right)\right) = \frac{5}{12}$$

So, It converges because the limit of the partial sum converges.
And it does so to $5/12$.

## Harmonic Series

$\sum_{n=1}^{\infty} \frac{1}{n}$ The Harmonic Series, diverges