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Series:

Let's say we have a sequence $\{a_n\}_{n=1}^{\infty}$. Let's now construct a new one:

Let's call these terms S's: $S_1 = a_1$

$S_2 = a_1 + a_2$

$S_3 = a_1 + a_2 + a_3$

$\vdots$

$S_N = a_1 + a_2 + \ldots + a_{N-1} + a_N$

Then we have a sequence called a sequence of partial sums: $\{S_N\}_{N=1}^{\infty}$

The question is now, does this new 'sum' sequence converge?

In the same way we did for the other sequences, we take limits $\lim_{N\to\infty} S_N$

$$S_N = a_1 + a_2 + \ldots + a_N = \sum_{i=1}^{N} a_i \quad \text{[finite series]}$$

That's what we are actually doing, knowing the result of this summation/series.

$$\underbrace{\lim_{N\to\infty} S_N = \lim_{N\to\infty} \sum_{i=1}^{N} a_i = \sum_{i=1}^{\infty} a_i}_{\text{Definition of an infinite series!}}$$

So, the question "does the seq. of partial sums converge?" is equivalent to asking "does $\sum_{i=1}^{\infty} a_i$ converge?".

FACTS:

- If $\sum a_i$ and $\sum b_i$ both converge:
  - (a) $\sum c a_i$ converges and it's equal to $c \cdot \sum a_i$
  - (b) $\sum a_i \pm b_i$ converges and it's equal to $\sum a_i \pm \sum b_i$

NOTATION:

$$\sum_{i=2}^{\infty} b_i$$

i → Index of Summation

Ex.:

$$\sum_{i=2}^{\infty} \frac{i+1}{i^2-1} = \underset{i=2}{\frac{3}{3}} + \underset{i=3}{\frac{4}{8}} + \underset{i=4}{\frac{5}{15}} + \ldots$$

↓ nothing sacred about choosing i, or n, or K...

INDEX SHIFT

Ex.: $\sum_{n=0}^{\infty} \frac{n+4}{2^n}$ Let's say I want to rewrite this as a series that starts at $n=3$ instead of $n=0$ but that has exactly the same value.

$$\sum_{i=3}^{\infty} \frac{(i-3)+4}{2^{i-3}} = \sum_{i=3}^{\infty} \frac{i+1}{2^{i-3}} = \sum_{n=3}^{\infty} \frac{n+1}{2^{n-3}}$$

So if I start writing the terms ($n=0$ and $i=3$), they will be completely equal.

$i=3$ → works provided $i=n+3$, then $n=i-3$

"You take back from n what you moved forward for your index n"

Ex: Write $\sum_{n=1}^{\infty} ar^{n-1}$ as a series that starts at $n=0$

$$= \sum_{n=0}^{\infty} ar^{(n+1)-1} = \sum_{n=0}^{\infty} ar^{n}$$ [Remember this]