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MORE SEQUENCES:

Given a sequence $\{a_n\}$

① The sequence is called "INCREASING" if $a_n < a_{n+1}$ for all n.

② The sequence is called "DECREASING" if $a_n > a_{n+1}$ for all n.

③ If $\{a_n\}$ is an increasing or decreasing sequence, then $\{a_n\}$ is called MONOTONIC.

④ The sequence is called bounded above if $a_n \leq M$ for all n.

↳ NOTE: given an UPPER BOUND, any number above it is also an upper bound.

⑤ The sequence is called bounded below if $a_n \geq m$ for all n.

⑥ The sequence is called BOUNDED if it's bounded above and below.

$\exists m, M$ s.t. $m \leq a_n \leq M$ for all n.

EX:

(a) $\{-n^2\}_{n=0}^{\infty}$

- Note, if starting at $n=1$, every a_n would be negative, then $a_n < 0$ for all n. But since we have zero.

$a_n \leq 0$ So $\{a_n\}$ is bounded above by zero.

- Note: $\lim_{n\to\infty}(-n^2) = -\infty$. There are no lower bounds, no $\{a_n\}$ it's not bounded below.

Then $\{a_n\}$ is not BOUNDED.

- Let n be a non-neg int. Then $n < n+1$

$$n^2 < (n+1)^2 \quad \text{mult. by } -1$$

$$-n^2 > -(n+1)^2$$

$$a_n > a_{n+1}$$

So, the sequence is decreasing. Also, monotonic.

(b) $\{(-1)^n\}_{n=1}^{\infty} = \{-1, 1, -1, 1, -1, 1, \ldots\}$

- Notice $\{a_n\}$ is bounded above by 1, and below by -1.
Then $\{a_n\}$ is BOUNDED.

- Note that it isn't neither decreasing nor increasing.

$a_2 > a_3$, $a_3 < a_4$ So it's not monotonic.

(c) $\left\{ \frac{2}{n^2} \right\}_{n=1}^{\infty}$

- Let n be a positive integer.

$$n < n+1$$
$$n^2 < (n+1)^2$$
$$1 < \frac{(n+1)^2}{n^2}$$
$$\frac{1}{(n+1)^2} < \frac{1}{n^2} \Longrightarrow \frac{2}{(n+1)^2} < \frac{2}{n^2} \Longrightarrow a_{n+1} < a_n$$

So, (a_n) is decreasing. It's also monotonic.

In case like this, it's allowed to say it's clearly decreasing.

- Note (a_n) has a lower bound at zero because a_n is always positive.
- Upper bound at 2 because it's a decreasing seqeunce, therefore the first term is the largest term.
- So $\{a_n\}$ is BOUNDED.

<u>EX:</u>

$\left\{ \frac{n}{n+1} \right\}_{n=1}^{\infty}$

- Notice, it's not clear to see whether this seq. increases or decreases.

Let's remember what differentiation was for in Calc. I. If we're given:

$$f(x) = \frac{x}{x+1} \longrightarrow f'(x) = \frac{1}{(x+1)^2} > 0 \quad \text{for } x > -1.$$

Then, since $f'(x) > 0$ for all x, then a_n is increasing.

Rem.: Critical points, if DER $= 0$ or DNE, but it has to be in the domain.

- Lower bound: NOTICE the sequence it's always positive, then 0 is a lower bound. 1/2 is also a lower bound, because it's the 1st term and this is an increasing sequence.
- Upper bound: notice $n+1$ is always greater than n, then $\frac{n}{n+1} < 1$ always. then 1 is an upper bound.
- The sequence is BOUNDED.