03-28-2019

Ex: Find interval and radius of convergence:

$$\sum_{n=1}^{\infty} \frac{(-1)^n n}{4^n}(x+3)^n$$

we have conv. at $x=-3$

$|x-a|<R$ conv.

$|x-a|>R$ div.

→ Not even a guaranteed alternating series...

Ratio Test:

$$L = \lim_{n\to\infty}\left|\frac{(-1)^{n+1}(n+1)(x+3)^{n+1}}{4^{n+1}} \cdot \frac{4^n}{(-1)^n n (x+3)^n}\right|$$

$$= \lim_{n\to\infty}\left|\frac{(n+1)(x+3)}{4n}\right|$$

$$= \lim_{n\to\infty}\frac{(n+1)|x+3|}{4n} = |x+3| \lim_{n\to\infty}\frac{n+1}{4n} = |x+3|\frac{1}{4}$$

$L = \frac{1}{4}|x+3|$ What values of x make $L<1$.

$\frac{1}{4}|x+3|<1$ convergence.

$\frac{1}{4}|x+3|>1$ divergence.

$|x+3|<4$ ⟶ $-4<x+3<4$

Radius of 4, centered at -3 $\boxed{-7<x<1}$ ← No changes to

→ Check endpoints:

$x=-7$ ⟶ $\sum_{n=1}^{\infty}\frac{(-1)^n n}{4^n}(-7+3)^n = \sum_{n=1}^{\infty}\frac{(-1)^n n}{4^n}(-4)^n$

$= \sum_{n=1}^{\infty}\frac{(-1)^n \cdot n (-1)^n (4)^n}{4^n} = \sum_{n=1} n$ diverges

$x=1$ ⟶ $\sum_{n=1}^{\infty}\frac{(-1)^n n}{4^n}(1+3)^n = \sum_{n=1}^{\infty}\frac{(-1)^n n}{4^n}(4)^n = \sum_{n=1}^{\infty}(-1)^n n$ div. $\lim_{n\to\infty}(-1)^n n = DNE$

Ex: Find interval and radius of convergence.

$$\sum_{n=1}^{\infty}\frac{2^n}{n}(4x-8)^n$$

Ratio:

$$L = \lim_{n\to\infty}\left|\frac{2^{n+1}(4x-8)^{n+1}}{n+1}\cdot\frac{n}{2^n(4x-8)^n}\right|$$

$4\frac{15}{8}-8$

$\frac{15}{2}-\frac{16}{2}=-\frac{1}{2}$

$$= \lim_{n\to\infty}\left|\frac{(4x-8)2n}{n+1}\right| = |4x-8|\lim_{n\to\infty}\frac{2n}{n+1}$$

$L = 2|4x-8| < 1$ conv

$8|x-2|<1$ ⟶ $|x-2|<1/8 = R$

Interv. of converg.: $15/8 < x < 17/8$ → $\boxed{\frac{15}{8}\le x<\frac{17}{8}}$

→ Check endpoints:

$x=15/8$ ⟶ $\sum_{n=1}^{\infty}\frac{2^n}{n}\left(-\frac{1}{2}\right)^n = \sum_{n=1}^{\infty}\frac{2^n}{n}\frac{(-1)^n}{2^n} = \sum_{n=1}^{\infty}(-1)^n\frac{1}{n}$ converg. by Alternating Series test.

$x=17/8$ ⟶ $\sum_{n=1}^{\infty}\frac{2^n}{n}\left(\frac{1}{2}\right)^n = \sum_{n=1}^{\infty}\frac{2^n}{n}\cdot\frac{1^n}{2^n} = \sum_{n=1}^{\infty}\frac{1}{n}$ diverges (harmonic)

Ex: $\sum_{n=0}^{\infty} n!(2x+1)^n$ notice $a=-\frac{1}{2}$.

$$L=\lim_{n\to\infty}\left|\frac{(n+1)!(2x+1)^{n+1}}{n!(2x+1)^n}\right| = \lim_{n\to\infty} n+1\,|2x+1|$$

$$L = |2x+1| \lim_{n\to\infty} n+1 = \infty \quad (\text{provided } x \neq -\tfrac{1}{2})$$

won't affect L since n+1 ↑ except if $x=-\frac{1}{2}$

This tells me: divergence if $L=\infty>1$, which happens for all x except $x=-\frac{1}{2}$.

But we already know that at $x=-\frac{1}{2}$ (center) we converge.

Interval of convergence: $x=-\frac{1}{2}$ (not really an interval, 1-element set).

Radius: 0 (zero).