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HW Help-Session

#5 $\sum_{n=1}^{\infty} \frac{n^2 \sin^2(1-n)}{n^4+6}$ $\quad 0 \le \sin^2(1-n) \le 1$

Guess it convergences. Find a larger one that will converge.

- $\frac{n^2\sin^2(1-n)}{n^4+6} > 0$
- decreasing

$$\frac{n^2 \sin^2(1-n)}{n^4+6} \le \frac{n^2}{n^4+6} \le \frac{n^2}{n^4} = \frac{1}{n^2}$$

And we know that $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges (p-series test).

Thus, by Comparison Test, $\sum \frac{n^2 \sin^2(1-n)}{n^4+6}$ will converge.

#7 $\sum_{n=0}^{\infty} \frac{\cos(n\pi)}{n^2+1}$ where $\cos(n\pi) = \{1, -1, 1, -1, 1, -1 \ldots\} = (-1)^n$

$= \sum_{n=0}^{\infty} \frac{(-1)^n}{n^2+1}$ So, it's an alternating series, actually.

#4 $\sum_{n=1}^{\infty} \frac{3}{8^n}$

#8 $\sum_{n=0}^{\infty} \frac{1}{(-2)^{n+2}(n^2+1)} = \sum_{n=0}^{\infty} \frac{1}{(2)^{n+2}(-1)^{n+2}(n^2+1)} = \sum_{n=0}^{\infty} \frac{(-1)^{n+2}}{2^{n+2}(n^2+1)}$ Now it's alternating!

$\sum_{n=1}^{\infty} \frac{(3n-5)!}{(n+4)!}$ positive, decreasing. Ratio Test:

$$L = \lim_{n\to\infty} \left| \frac{(3n-2)!}{(n+5)!} \cdot \frac{(n+4)!}{(3n-5)!} \right|$$

$$= \lim_{n\to\infty} \left| \frac{\cancel{(n+4)!}}{(n+5)\cancel{(n+4)!}} \cdot \frac{(3n-2)(3n-3)(3n-4)\cancel{(3n-5)!}}{\cancel{(3n-5)!}} \right|$$

$$= \lim_{n\to\infty} \frac{(3n-2)(3n-3)(3n-4)}{(n+5)} \quad \begin{matrix} \text{deg} = 3 \\ \text{deg} = 1 \end{matrix}$$

$= \infty > 1$ Thus, the series converges.

#6 $\lim_{n\to\infty} ne^{-n} = \lim_{n\to\infty} \frac{n}{e^n} \to$ L'HôP.

Power Series:

→ might determine conv/div.

variable (fxn)

→ They're of the form $\displaystyle\sum_{n=0}^{\infty} c_n(x-a)^n = f(x)$ We can see them as functions

changing coefficients

center of the series

Assume convergence and thus also that series are "big giant" sums.

$$\sum_{n=0}^{\infty} c_n(x-a)^n = c_0 + c_1(x-a) + c_2(x-a)^2 + c_3(x-a)^3 + \dots$$

Observe that if we let $x=a$, then $\sum_{n=0}^{\infty} c_n(x-a)^n = c_0$

i.e. the series will converge where $x=a$ (no matter what).

When we can find a $R>0$ so that $|x-a|<R$ converges

and $|x-a|>R$ diverges

Then we call R the radius of convergence.

The interval of convergence is the set of all x's for which the power series converges.

Ex: We found R for a series. So we know $|x-a|<R$ converg & $|x-a|>R$ diverg.

$$-R < x-a < R$$
$$a-R < x < a+R$$

Then our interval of convergence is given by $(a-R, a+R)$. Although I don't know yet what happens at the edges $a-R$, $a+R$.