03-25-2019

Absolute Convergence:

Ex: Show the following series converges. $\sum_{n=2}^{\infty} \frac{\sin(n)}{n^3}$

→ Most tests require positive terms, so can't use them here.

→ Does it converge absolutely?

$$\sum_{n=2}^{\infty} \left| \frac{\sin(n)}{n^3} \right| = \sum_{n=2}^{\infty} \frac{|\sin(n)|}{n^3}$$ → Can't use Integr. test. (not monotone decr.)

→ Comparison test! $0 \le |\sin(n)| \le 1$

Then $\sum_{n=2}^{\infty} \frac{|\sin(n)|}{n^3} \le \sum_{n=2}^{\infty} \frac{1}{n^3}$ → converges by p-series test $(p=3>1)$

So, by comparison test, $\sum_{n=2}^{\infty} \frac{|\sin(n)|}{n^3}$ converges.

Thus, $\sum_{n=2}^{\infty} \frac{\sin(n)}{n^3}$ converges absolutely, and so it converges.

Ratio Test:

Given a series $\sum a_n$, compute:

$L = \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right|$. Then
1. If $L<1$, the series is absolutely convergent.
2. If $L>1$, then the series is divergent.
3. If $L=1$, we don't know.

↳ $(L \geq 0)$

Ex: $\sum_{n=1}^{\infty} \frac{(-10)^n}{4^{2n+1}(n+1)}$ → we can treat it as an alternating series ✗.

$= \sum \frac{(-1)^n 10^n}{4^{2n+1}(n+1)}$ but then checking that it decreases is a lot of work.

→ Ratio Test:

$$a_n = \frac{(-10)^n}{4^{2n+1}(n+1)} \qquad a_{n+1} = \frac{(-10)^{n+1}}{4^{2n+2+1}(n+2)}$$

$$L = \lim_{n\to\infty} \left| \frac{(-10)^{\cancel{n}+1}}{4^{\cancel{2n+3}}(n+2)} \cdot \frac{4^{\cancel{2n+1}}(n+1)}{\cancel{(-10)^n}} \right| = \lim_{n\to\infty} \left| \frac{(-10)^1 (n+1)}{4^2 (n+2)} \right|$$

$$= \lim_{n\to\infty} \frac{10(n+1)}{16(n+2)} = \frac{10}{16} < 1$$

Then the series is absolutely convergent, so it converges.

FACTORIALS:

$n! = n(n-1)(n-2)\cdots(3)(2)(1)$ $\qquad 0! = 1$ (by definition)

$1! = 1$

$2! = 2\cdot1 = 2$

$3! = 3\cdot2\cdot1 = 6$

$4! = 4\cdot3\cdot2\cdot1 = 24$

$5! = 5\cdot4\cdot3\cdot2\cdot1 = 120$

$6! = 6\cdot5\cdot4\cdot3\cdot2\cdot1 = 720$
$= 6\cdot5!$
$= 6\cdot5\cdot4!$
$= 6\cdot5\cdot4\cdot3!$

$(n+1)! \neq n! + 1!$

$(2n)! \neq 2\cdot n!$

Ex: $\sum_{n=0}^{\infty} \frac{n!}{5^n}$

$$L = \lim_{n\to\infty} \left| \frac{(n+1)!}{5^{n+1}} \cdot \frac{5^n}{n!} \right|$$

$$= \lim_{n\to\infty} \left| \frac{(n+1)\cancel{n!}\,\cancel{5^n}}{\cancel{5^n}\cdot 5 \cdot \cancel{n!}} \right| = \lim_{n\to\infty} \frac{n+1}{5} = \frac{1}{5}\lim_{n\to\infty}(n+1) = \infty$$

Since $\infty > 1$, then the series diverges.