03-20-2019

Integral Test:

• **Ex**: Converge or diverge?

• $\sum_{n=0}^{\infty} ne^{-n^2}$ Note the following: $ne^{-n^2} > 0$ for $n \geq 1$ (eventually)

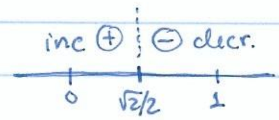
→ It's not obvious if it's decreasing or increasing.

$$f(x) = xe^{-x^2}$$

$$f'(x) = e^{-x^2} + xe^{-x^2}(-2x)$$

$$= e^{-x^2} - 2x^2e^{-x^2}$$

$$= e^{-x^2}(1-2x^2) = 0$$



$$1-2x^2 = 0$$

$$\frac{1}{2} = x^2 \rightarrow \pm\sqrt{\frac{1}{2}} = \pm\frac{\sqrt{2}}{2}$$ (negat. is out of domain)

So, it's eventually decreasing.

Then we can do integral test!

$\rightarrow \int_0^{\infty} xe^{-x^2}dx \quad u = -x^2 \quad du = -2x\,dx \quad dx = \frac{du}{-2x}$

$$\lim_{b\to\infty} \int_0^{\infty} xe^{u}\frac{du}{-2x} = \lim_{b\to\infty}\left(\frac{-1}{2}\right)\int_0^{\infty} e^{u}du = -\frac{1}{2}\left[\lim_{b\to\infty}\left[e^{-x^2}\right]_0^b\right]$$

$$= -\frac{1}{2}\left[\lim_{b\to\infty}\left(e^{-b^2} - \overset{1}{\cancel{e^0}}\right)\right] = \text{converges.}$$

So, by Integral test, $\sum_{n=0}^{\infty} ne^{-n^2}$ also converges.

• **FACT**: $p > 0$

$\sum_{n=1}^{\infty} \frac{1}{n^p}$ Notice $\frac{1}{n^p} > 0$ and it decreases. Then we can do Integral Test:

$$\int_1^{\infty} \frac{1}{x^p}dx = \begin{cases} \text{converges if } p > 1 \\ \text{diverges if } 0 < p \leq 1 \end{cases}$$

So, the series will converge if $p > 1$, and diverge if $p \leq 1$.

Comparison Test / Limit Comparison Test:

Comparison Test:

Given two series $\sum a_n$ and $\sum b_n$ with $a_n, b_n \geq 0$ for all n (eventually).

Further, assume $a_n > b_n$:

1. If $\sum a_n$ converges, then $\sum b_n$ converges too.
2. If $\sum b_n$ diverges, then $\sum a_n$ diverges as well.

· **Ex**: Converge or diverge?

$$\sum_{n=0}^{\infty} \frac{1}{3^n+n}$$ → Note $\frac{1}{3^n+n} > 0$ always.

We're going to guess it'll converge. Then we need a larger convergent series.

Notice that $\frac{1}{3^n+n} < \frac{1}{3^n}$

Where $\sum_{n=0}^{\infty} \frac{1}{3^n}$ looks a lot like a geometric series $\sum_{n=0}^{\infty} ar^n$

$$\sum_{n=0}^{\infty} \frac{1}{3^n} = \sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^n$$ where $|\frac{1}{3}| < 1$, then it converges!

$$= \frac{1}{1-(1/3)} = 3/2$$

Then, by Comparison test, $\sum_{n=0}^{\infty} \frac{1}{3^n+n}$ converges!

· **Ex**: Converge or diverge?

$$\sum_{n=1}^{\infty} \frac{n}{n^2-\cos^2(n)}$$ → Note that $\frac{n}{n^2-\cos^2(n)} > 0$, then we can do it.

We're going to guess it'll diverge. Let's find a smaller divergent.