03-18-2019

HW help Session

#9 (similar):

- $\left\{\frac{7-n}{3+8n}\right\}_{n=1}^{\infty}$ Note that $f(x) = \frac{7-x}{3+8x} \rightarrow f'(x) = \frac{-59}{(3+8x)^2} < 0$, decreasing

also monotone

→ Since it's decreasing, it's first term is an upper bound. Bounded above by 6/11.

→ Bounded below?

If we take the limit: $\lim_{n\to\infty} \frac{7-n}{3+8n} = \frac{-1}{8}$.

Suppose $\frac{-1}{8}$ is not a lower bound. Then there's a seq. term below it.
But since my limit is $-\frac{1}{8}$, the terms would have to increase at some point. But this would contradict the fact that this a decreasing sequence. CONTRADICTION.
Means our assumption is false. Then $-\frac{1}{8}$ is a lower bound.

→ We have two bounds (above, below), then the whole sequence is bounded.

Telescoping series:

- $\sum_{n=3}^{\infty} \frac{4}{2n-n^2} = \sum_{n=3}^{\infty} \frac{4}{n(2-n)}$ then $S_N = \sum_{i=3}^{N} \frac{4}{i(2-i)} = \sum_{i=3}^{N} \frac{2}{i} - \frac{2}{i-2}$ Telescoping!

$$S_N = \left(\cancel{\frac{2}{3}} - \frac{2}{1}\right) + \left(\cancel{\frac{2}{4}} - \frac{2}{2}\right) + \left(\cancel{\frac{2}{5}} - \cancel{\frac{2}{3}}\right) + \left(\frac{2}{6} - \cancel{\frac{2}{4}}\right) +$$

$$+ \left(\frac{2}{7} - \cancel{\frac{2}{5}}\right) + \dots + \left(\cancel{\frac{2}{N-5}} - \frac{2}{N-7}\right) + \left(\cancel{\frac{2}{N-4}} - \frac{2}{N-6}\right) +$$

$$+ \left(\cancel{\frac{2}{N-3}} - \cancel{\frac{2}{N-5}}\right) + \left(\cancel{\frac{2}{N-2}} - \cancel{\frac{2}{N-4}}\right) + \left(\frac{2}{N-1} - \cancel{\frac{2}{N-3}}\right) + \left(\frac{2}{N} - \cancel{\frac{2}{N-2}}\right)$$

$$S_N = -2 - 1 + \frac{2}{N-1} + \frac{2}{N}$$

$$\lim_{N\to\infty} S_N = \lim_{N\to\infty}\left(-3 + \frac{2}{N-1} + \frac{2}{N}\right) = -3$$

Then:

$$\sum_{n=3}^{\infty} \frac{4}{2n-n^2} = -3$$

#7 (Similar): Given: $\sum_{n=5}^{\infty} \frac{n^2}{1+7n^4} = 0.03162$, then what's $\sum_{n=2}^{\infty} \frac{n^2}{1+7n^4}$ equal to?

$$\sum_{n=2}^{\infty} \frac{n^2}{1+7n^4} = \overset{n=2}{\frac{4}{1+7\cdot 2^4}} + \overset{n=3}{\frac{9}{1+7\cdot 3^4}} + \overset{n=4}{\frac{16}{1+7\cdot 4^4}} + \sum_{n=5}^{\infty} \frac{n^2}{1+7n^4}$$

$$= 0.06017 + 0.03162$$

$$= 0.09179$$

#9 (similar). Converge or Diverge? $\sum_{n=0}^{\infty} a_n$ given $S_N = \dfrac{1+8N^2}{N-4}$ as the partial sum term.

Then we need to take the limit:

$$\lim_{N\to\infty} S_N = \lim_{N\to\infty} \frac{1+8N^2}{N-4} = \infty$$

since:

$$\lim_{N\to\infty} S_N = \sum_{n=0}^{\infty} a_n . \quad \text{Then } \sum_{n=0}^{\infty} a_n \text{ diverges.}$$

If $S_N = \dfrac{1+8N}{N-4}$, then $\sum_{n=0}^{\infty} a_n$ would converge.

Also remember:

if the series was $\sum_{n=5}^{\infty} \dfrac{1+8N}{N-4}$, then it'd diverge too since $\lim_{n\to\infty} a_n \neq 0$.

#10 (similar)

- $\left\{ \dfrac{10+n}{25000+3n^2} \right\}_{n=0}^{\infty}$ Note all terms are positive, then bounded below by zero.

If $f(x) = \dfrac{10+x}{25000+3x^2} \longrightarrow f'(x) = \dfrac{25000-60x-3x^2}{(25000+3x^2)^2}$

Now, find the critical points: $25000-60x-3x^2=0$

$$x = \frac{60 \pm \sqrt{60^2-4(-3)(25000)}}{2(-3)} = \begin{matrix} -101.8332 \\ 81.8332 \end{matrix}$$

outside domain (-101.8332)

$f'(81)>0$ $f'(82)<0$

$\longleftarrow$ INCR. | DECR. $\longrightarrow$

81.8332

Note, at $x = 81.8332$ would give us an upper bound.

$f(81.8332) =$ __________ is an upper bound.

Another way: observe that the numerator is way smaller than the denominator. This give us a free upper bound, 1.