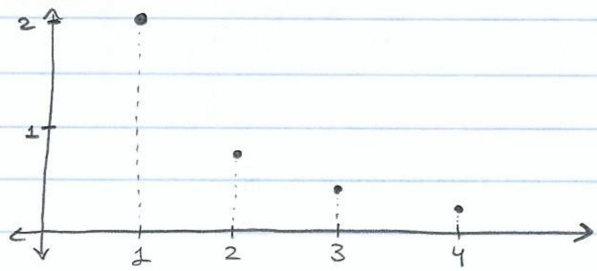
03-01-2019

Sequences:

Ex: $\left\{ \frac{n+1}{n^2} \right\}_{n=1}^{\infty} = \left\{ 2, \frac{3}{4}, \frac{4}{9}, \frac{5}{16}, \ldots \right\}$

"Sketch"



→ Observe it's gonna go to zero.

→ We can take limits of sequences also.

$$\lim_{n\to\infty} \frac{n+1}{n^2} = 0 \quad \text{(intuition, lim techniques...)}$$

→ So, for a given sequence $\{a_n\}$ we can have:

- $\lim_{n\to\infty} \{a_n\}$ exists and is finite. ⟶ the sequence is CONVERGENT
- $\lim_{n\to\infty} \{a_n\}$ doesn't exist or is infinite. ⟶ the sequence is DIVERGENT

→ Properties & Facts of sequences (limits):

#1 Given a function $\{a_n\}$ and a function $f(x)$ so that $f(n) = a_n$ for all n, then if $\lim_{x\to\infty} f(x) = L$, then $\lim_{n\to\infty} a_n = L$.

#2 Given a sequence $\{a_n\}$, if $\lim_{n\to\infty} |a_n| = 0$, then $\lim_{n\to\infty} a_n = 0$.

$\{a_n\}$ and $\{b_n\}$ are both convergent sequences. Then:

1. $\lim_{n\to\infty} (c a_n) = c \lim_{n\to\infty} (a_n)$
2. $\lim_{n\to\infty} (a_n \pm b_n) = \left(\lim_{n\to\infty} a_n\right) \pm \left(\lim_{n\to\infty} b_n\right)$
3. $\lim_{n\to\infty} (a_n b_n) = \left(\lim_{n\to\infty} a_n\right)\left(\lim_{n\to\infty} b_n\right)$
4. $\lim_{n\to\infty} \left(\frac{a_n}{b_n}\right) = \frac{\lim_{n\to\infty} a_n}{\lim_{n\to\infty} b_n}$, provided $\lim_{n\to\infty} b_n \neq 0$.
5. $\lim_{n\to\infty} (a_n)^p = \left(\lim_{n\to\infty} a_n\right)^p$

<u>Ex</u>: Converge or diverge? Limiting value if convergent.

(a) $\left\{ \dfrac{6-3n^2}{4n^2+7n-1} \right\}_{n=12}^{\infty}$

$$f(x) = \frac{6-3x^2}{4x^2+7x-1} \rightarrow \lim_{x\to\infty} \underbrace{\frac{6-3x^2}{4x^2+7x-1}}_{\text{factor out the highest degree of the denominator...}} = \lim_{x\to\infty} \frac{\cancel{x^2}(\overset{0}{\overset{\uparrow}{\frac{6}{x^2}}}-3)}{\cancel{x^2}(4+\underset{\underset{0}{\downarrow}}{\frac{7}{x}}-\underset{\underset{0}{\downarrow}}{\frac{1}{x^2}})} = \frac{-3}{4}$$

So, this sequence converges. Its limiting value is $\frac{-3}{4}$.

(b) $\left\{ \dfrac{e^{2n}}{n} \right\}_{n=4}^{\infty}$

$$f(x) = \frac{e^{2x}}{x} \rightarrow \lim_{x\to\infty} \frac{e^{2x}}{x} = \frac{\infty}{\infty} \quad \text{L'HÔPITALS}$$

$$\lim_{x\to\infty} \frac{[e^{2x}]'}{[x]'} = \lim_{x\to\infty} \frac{2e^{2x}}{1} = \infty$$

This sequence diverges (to ∞).

(c) $\left\{ \dfrac{(-1)^n}{n} \right\}_{n=1}^{\infty}$

$\lim\limits_{n\to\infty} \dfrac{(-1)^n}{n}$ We never learned (in Calc 1) how to take limits of alternating functions, terms. Here's where we use the abs. val. property.

$$\lim_{n\to\infty} \left| \frac{(-1)^n}{n} \right| = \lim_{n\to\infty} \frac{|(-1)^n|}{|n|} = \lim_{n\to\infty} \frac{1}{n} = 0$$

By fact #2 this means $\lim\limits_{n\to\infty} \dfrac{(-1)^n}{n} = 0$.

(d) $\{(-1)^n\}_{n=0}^{\infty} = \{1, -1, 1, -1, \ldots\}$

$\lim\limits_{n\to\infty} (-1)^n$

"does not exist"

Only when we know the terms will always alternate/oscillate we can justify our answer writing down first terms and that it DIVERGES.