02-08-2019

# HW-HELP SESSION

**Ex**: Similar to #

- $\int e^{12x}\sqrt{e^{6x}-9}\,dx$

we want $\sqrt{(\ )^2-9}$. Then we choose $u=e^{3x} \longrightarrow u^3=e^{9x}$

$du = 3e^{3x}dx$

$dx = \frac{du}{3e^{3x}}$

$$= \int u^3\sqrt{u^2-9}\,\cancel{e^{3x}}\frac{du}{3\cancel{e^{3x}}}$$

$$= \frac{1}{3}\int u^3\sqrt{u^2-9}\,du \qquad u=3\sec\theta \quad du=3\sec\theta\tan\theta\,d\theta$$

$\sqrt{u^2-9}$
$= \sqrt{9\sec^2\theta-9} = \sqrt{9(\sec^2\theta-1)} = 3|\tan\theta|$ indef integ
$3\tan\theta$

$$= \frac{1}{\cancel{3}}\int 27\sec^3\theta\cdot\cancel{3}\tan\theta\cdot 3\sec\theta\tan\theta\,d\theta$$

$$= 81\int \sec^4\theta\tan^2\theta\,d\theta$$

$$= 81\int \sec^2\theta\tan^2\theta\sec^2\theta\,d\theta \qquad v=\tan\theta$$

$$= 81\int(\tan^2\theta+1)\tan^2\theta\sec^2\theta\,d\theta \qquad dv=\sec^2\theta\,d\theta$$

$$= 81\int(v^2+1)v^2\,dv \qquad [\ldots]$$

**Ex**: Similar to #7

- $\int_1^4 \frac{1}{y^2-3y}dy = \int_1^3 \frac{1}{y^2-3y}dy + \int_3^4\frac{1}{y^2-3y}dy$


$$= \lim_{t\to3^-}\int_1^t \frac{1}{y^2-3y}dy + \lim_{s\to3^+}\int_s^4\frac{1}{y^2-3y}dy$$

Now by partial fraction...

$$\frac{1}{y(y-3)} = \frac{A}{y}+\frac{B}{y-3}$$

$$= Ay-3A+By$$

$$1 = y(A+B)-3A$$

$A+B=0$ $\quad -3A=1$

$A=-B$ $\quad \boxed{A=-1/3}$

$\boxed{B=1/3}$

$$= \lim_{t\to3^-}\frac{1}{3}\int_1^t -\frac{1}{y}+\frac{1}{y-3}dy + \lim_{s\to3^+}\frac{1}{3}\int_s^4 -\frac{1}{y}+\frac{1}{y-3}dy$$

$$= \frac{1}{3}\lim_{t\to3^-}\Big[-\ln|y|+\ln|y-3|\Big]_1^t + \frac{1}{3}\lim_{s\to3^+}\Big[-\ln|y|+\ln|y-3|\Big]_s^4$$

$$= \frac{1}{3}\lim_{t\to3^-}\Big[-\ln|t|+\underbrace{\ln|t-3|}_{-\infty}+\ln|1|-\ln|1-3|\Big]$$

$$+\frac{1}{3}\lim_{s\to3^+}\Big[-\ln|4|+\ln|4-3|+\ln|s|-\underbrace{\ln|s-3|}_{+\infty}\Big]$$

DIVERGENT!

So, the original integral is divergent.

Ex: Similar to #3

- $\int \frac{(1+\cos x)\sin x}{\cos^2 x - 14\cos x + 24}\,dx$  $u=\cos x$, $du = -\sin x\,dx$

$= -\int \frac{(1+u)}{u^2-14u+24}\,du = -\int \frac{1+u}{(u-12)(u-2)}\,du$  Now go on with part. frac. decomp.

$\begin{matrix} u & -12 \\ u & -2 \end{matrix}$

if the problem was $= -\int \frac{1+u^2}{u^2-14u+24}\,du$  Then long division.

$$\begin{array}{r|l} u^2+0u+1 & u^2-14u+24 \\ -\,u^2-14u+24 & 1 \\ \hline 0\quad 14u-23 & \end{array}$$

$= -\int 1 + \frac{14u-23}{(u-12)(u-2)}\,du$ → partial fr. again...

Ex: Similar to #1

- $\int \cos^4(4t)\sin^3(8t)\,dt$  using trig ident.: $\sin(2x) = 2\sin(x)\cos(x)$

$= \int \cos^4(4t)(\sin(8t))^3\,dt$

$= \int \cos^4(4t)(2\sin(4t)\cos(4t))^3\,dt$ ... now all have same argument.

Ex: Similar to #9

- $\int_1^\infty \frac{xe^{-x}-\sin^2 x}{x^4}\,dx = \underbrace{\int_1^\infty \frac{xe^{-x}}{x^4}\,dx}_{\boxed{1}} - \underbrace{\int_1^\infty \frac{\sin^2 x}{x^4}\,dx}_{\boxed{2}}$

$\boxed{1}$ $\int_1^\infty \frac{xe^{-x}}{x^4}\,dx = \int_1^\infty \frac{e^{-x}}{x^3}\,dx = \int_1^\infty \frac{1}{x^3e^x}\,dx \le \int_1^\infty \frac{1}{x^3}\,dx$ convergent

$\boxed{2}$ $\sin^2 x \le 1$

$\int_1^\infty \frac{\sin^2 x}{x^4}\,dx \le \int_1^\infty \frac{1}{x^4}\,dx$ converges

Thus, = convergent − convergent = convergent