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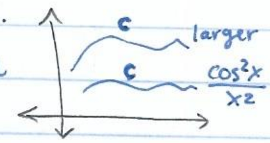
Ex:

- $\int_2^{\infty} \frac{\cos^2 x}{x^2}\,dx$. We'll compare it with $\int_a^{\infty} \frac{1}{x^p}\,dx$ for when $p>1$.
  Find a larger function for which the integral converges.



Let's remember: $0 \le \cos^2(x) \le 1$

$$\frac{\cos^2(x)}{x^2} \le \frac{1}{x^2}$$

- First attempt: since $x \ge 2$, then

$$\frac{\cos^2 x}{x^2} \le \frac{\cos^2 x}{4}$$

Let's see if $\int \frac{\cos^2 x}{4}\,dx$ converges...

$$= \lim_{t\to\infty} \int_2^t \frac{1}{8}(1+\cos(2x))\,dx = \ldots = \text{diverges! } ☹$$

- Second attempt: $\frac{\cos^2(x)}{x^2} \le \frac{1}{x^2}$

Where $\int \frac{1}{x^2}\,dx$ converges since it's of the type $\int \frac{1}{x^p}\,dx$ where $p>1$!

Therefore, by comparison test, since:

$$\int \frac{\cos^2(x)}{x^2}\,dx \le \int \frac{1}{x^2}\,dx \text{ convergent}$$

Then $\int \frac{\cos^2(x)}{x^2}\,dx$ also converges.

Ex:

- $\int_3^{\infty} \frac{1}{x+e^x}\,dx$ Note how the e^x grows much faster than x (in the denominator), making $x+e^x$ actually just be e^x.

Examples: $\frac{1}{x+1}$ DIVERG. $\quad \frac{1}{x+e^x}$ fast!!! $\quad \frac{1}{x+e^{-x}}$, $e^{-x} \to 0$, $\approx \frac{1}{x}$ DIVERGES

Looks like this integral will converge, let's find a larger fxn whose integral from 3 to ∞ converges.

$\frac{1}{x+e^x} \le (\quad) \rightarrow$ we're looking for something larger and convergent.
We can only mess w/the denominator. Let's play by taking out stuff from it.

Does $\int \frac{1}{x}\,dx$ (taking out e^x) converge? No, $p \le 1$!

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What about $\int \frac{1}{e^x}dx$?

$$\lim_{t\to\infty}\int_3^t e^{-x}\,dx = \left[-e^{-x}\right]_3^t = \lim_{t\to\infty}\left(\frac{-1}{e^t}^{\to 0} + \frac{1}{e^{-3}}\right) = e^{-3} \quad \text{converges!}$$

So we found $\frac{1}{x+e^x} \le \frac{1}{e^x}$ where $\int \frac{1}{e^x}$ converges.

Thus, By Compariso Test, $\int \frac{1}{x+e^x}dx$ converges as well.

Ex:

- $\int_3^\infty \frac{1}{x-e^{-x}}dx$. Let's guess it diverges. Then we need a smaller fxn whose integral diverges.

  e^{-x} ↳ goes to zero.

  $\frac{1}{x-e^{-x}} \ge \bigcirc$ We can make the denominator larger by not substracting e^{-x} from x.

  $\frac{1}{x-e^{-x}} \ge \frac{1}{x}$. Note that $\int_3^\infty \frac{1}{x}dx$ diverges.

  Then by comparison test, $\int_3^\infty \frac{1}{x-e^{-x}}dx$ also diverges.