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IMPROPER INTEGRALS (Cont'd)

Ex: $\displaystyle\int_0^3 \frac{1}{\sqrt{3-x}}\,dx$ Note, the problem here is at $x=3$.

$$= \lim_{t\to 3^-}\int_0^t (3-x)^{-1/2}\,dx = \lim_{t\to 3^-}\left[-2(3-x)^{1/2}\right]_0^t$$

0 3

we're looking to the left of 3

$$= \lim_{t\to 3^-}\left[-2(3-t)^{1/2}+2(3-0)^{1/2}\right]$$

$$t\to 3^-\left[-2\sqrt{3-t}+2\sqrt{3}\right]$$

$$\left[-2\sqrt{3-3}^{\,0}+2\sqrt{3}\right] = 2\sqrt{3}$$

Then, the original integral is convergent and its value is $2\sqrt{3}$.

Ex: Conv. or Div.? If conv., give value!

$$\int_{-2}^{3}\frac{1}{x^3}\,dx \overset{?}{=} \int_{-2}^{0}\frac{1}{x^3}\,dx + \int_{0}^{3}\frac{1}{x^3}\,dx$$

problem at zero

Only valid if both new integr. converge.

-2 0 3

$$= \lim_{t\to 0^-}\int_{-2}^{t} x^{-3}\,dx + \lim_{s\to 0^+}\int_{s}^{3} x^{-3}\,dx$$

$$= \lim_{t\to 0^-}\left[\frac{1}{-2x^2}\right]_{-2}^{t} + \lim_{s\to 0^+}\left[\frac{-1}{2x^2}\right]_{s}^{3}$$

$$= \lim_{t\to 0^-}\left[\frac{1}{-2t^2}+\frac{1}{8}\right] + \lim_{s\to 0^+}\left[-\frac{1}{18}+\frac{1}{2s^2}\right]$$

$$-\infty \qquad\qquad +\infty$$

DO NOT MAKE THE MISTAKE OF CANCELING THE INFINITIES

I can already say the integral is divergent and move on.

Comparison Test for Improper Integrals.

Ex: • $\displaystyle\int_2^\infty \frac{\cos^2(x)}{x}\,dx$ I still want to know if this diverges or converges. But who can integrate this!!??

If $f(x)\le g(x)$, then:

1. If $\int_a^\infty f(x)\,dx$ is divergent, then $\int_a^\infty g(x)\,dx$ has to be divergent too.
2. If $\int_a^\infty g(x)\,dx$ is convergent, then $\int_a^\infty f(x)\,dx$ is convergent too.

• $\displaystyle\int_2^\infty \frac{\cos^2(x)}{x^2}\,dx$. Let's remember $\int_a^\infty \frac{1}{x^p}\,dx \begin{cases}\text{conv if } p>1\\ \text{div if } p\le 1\end{cases}$

Now, note: $0\le \cos^2 x \le 1$

Then $\dfrac{\cos^2 x}{x^2} \le \dfrac{1}{x^2}$