02-27-2019

■ Similar to #6:

"$\cos^2\left(\frac{\theta}{2}\right) = r$" convert to Cartesian.

$\frac{1}{2}(1+\cos\theta) = r$

$1 + \cos\theta = 2r$

$r + r\cos\theta = 2r^2$ → $\sqrt{x^2+y^2} + x = 2(x^2+y^2)$

...

■ Similar to #4: *Compute total distance traveled.*

$x = \cos^2(t/3)$ for $-2\pi \le t \le 7\pi$

$y = 1 + 2\cos^4(t/3)$

• From previous problems: $0 \le t \le \frac{3\pi}{2}$ range of t for 1 trace

6 traces.

Length of one trace.

$L = \int ds \qquad ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

$= \sqrt{\left(-2\cos\left(\frac{t}{3}\right)\sin\left(\frac{t}{3}\right)\cdot\frac{1}{3}\right)^2 + \left(-8\cos^3\left(\frac{t}{3}\right)\sin\left(\frac{t}{3}\right)\cdot\frac{1}{3}\right)^2}\, dt$

$= \sqrt{\frac{4}{9}\cos^2\left(\frac{t}{3}\right)\sin^2\left(\frac{t}{3}\right) + \frac{64}{9}\cos^6\left(\frac{t}{3}\right)\sin^2\left(\frac{t}{3}\right)}\, dt$

$= \sqrt{\frac{4}{9}\cos^2\left(\frac{t}{3}\right)\sin^2\left(\frac{t}{3}\right)\left(1 + 16\cos^4\left(\frac{t}{3}\right)\right)}\, dt$

$= \frac{2}{3}\left|\cos\left(\frac{t}{3}\right)\right|\left|\sin\left(\frac{t}{3}\right)\right|\sqrt{1 + 16\cos^4(t/3)}\, dt$

$L = \int_0^{3\pi/2} \frac{2}{3}\left|\cos\left(\frac{t}{3}\right)\right|\left|\sin\left(\frac{t}{3}\right)\right|\sqrt{1+16\cos^4(t/3)}\, dt$

$0 \le t \le 3\pi/2$

$0 \le \frac{t}{3} \le \pi/2$

thus sin, cos are both posit. We can drop abs. val. bars

$= \int_0^{3\pi/2} \frac{2}{3}\cos\left(\frac{t}{3}\right)\sin\left(\frac{t}{3}\right)\sqrt{1+16\cos^4(t/3)}\, dt$

$= \int_0^{3\pi/2} \frac{2}{3}\cos\left(\frac{t}{3}\right)\sin\left(\frac{t}{3}\right)\sqrt{1 + \left(4\cos^2\left(\frac{t}{3}\right)\right)^2}\, dt$

$u = 4\cos^2\left(\frac{t}{3}\right)$

$du = -\frac{8}{3}\cos\left(\frac{t}{3}\right)\sin\left(\frac{t}{3}\right) dt$

$= -\frac{1}{4}\int_0^{3\pi/2} \sqrt{1+u^2}\, du$ Now trig-sub

$u = \tan\theta \to du = \sec^2\theta\, d\theta$

$\sqrt{1+u^2} = \sqrt{1+\tan^2\theta}$

$= |\sec\theta| = \sec\theta$

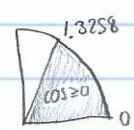
$= -\frac{1}{4}\int_4^0 \sqrt{1+u^2}\, du$

$0 \le t \le 3\pi/2$

$u = 4\cos^2\left(\frac{0}{3}\right) = 4$, $u = 4\cos^2\left(\frac{3\pi}{2}\right) = 0$

$= -\frac{1}{4}\int_{1.3258}^0 \sec\theta\sec^2\theta\, d\theta$

$u=4 \to 4 = \tan\theta \to \theta = \arctan(4) = 1.3258$ (rad)



$= -\frac{1}{4}\left(\frac{1}{2}\right)\left[\sec\theta\tan\theta + \ln|\sec\theta + \tan\theta|\right]_{1.3258}^{0}$

$u=0 \to 0 = \tan\theta \to \theta = \arctan(0) = 0$ (rad)

then $\cos\theta \ge 0$, $\sec\theta \ge 0$.

$= 2.3231$

Then we can drop abs. val. bars.

positive length

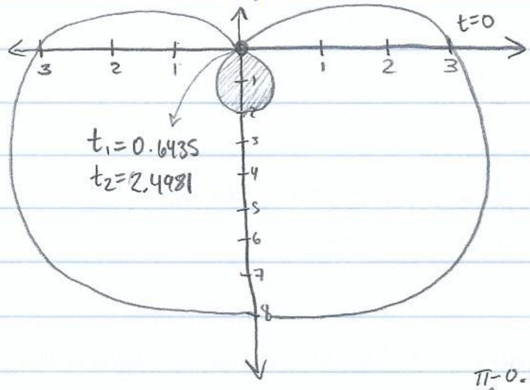
TOTAL DISTANCE TRAVELED:

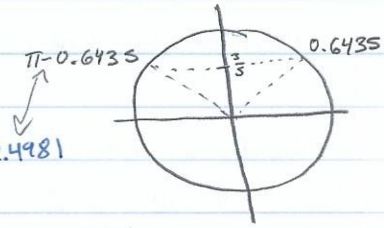
$2.3231 \times 6 = 13.9386$

■ Similar to #5:

$r = 3 - 5\sin\theta$, area of the inner loop.

θ	r
0	3
$\pi/2$	-2
π	3
$3\pi/2$	8
2π	3





When do we cross the origin?

$0 = 3 - 5\sin\theta \rightarrow \sin\theta = \frac{3}{5} \rightarrow \theta = 0.6435,\ 2.4981$

Thus,

$$A = \int_{0.6435}^{2.4981} \frac{1}{2}(3 - 5\sin\theta)^2\, d\theta$$

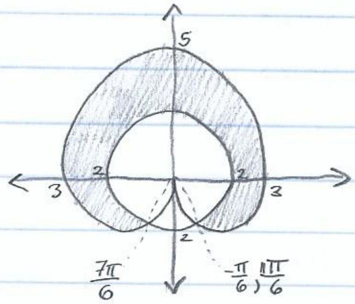
$$= \frac{1}{2}\int_{0.6435}^{2.4981} 9 - 30\sin\theta + \underbrace{25\sin^2\theta}_{\frac{25}{2}(1-\cos 2\theta)}\, d\theta$$

$$= \frac{1}{2}\int_{0.6435}^{2.4981} \frac{43}{2} - 30\sin\theta - \frac{25}{2}\cos(2\theta)\, d\theta = \frac{1}{2}\left[\frac{43}{2}\theta + 30\cos\theta - \frac{25}{4}\sin 2\theta\right]_{0.6435}^{2.4981}$$

$$= 1.9368$$

EX: Inside $r = 3 + 2\sin\theta$

Outside $r = 2$



$$A_1 = \int_{-\pi/6}^{7\pi/6} \frac{1}{2}\left[(3+2\sin\theta)^2 - (2)^2\right] d\theta$$

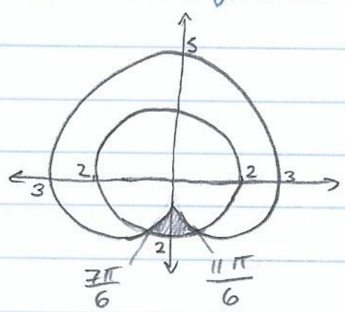
$$= \int_{-\pi/6}^{7\pi/6} \frac{1}{2}\left(9 + 4\sin^2\theta + 12\sin\theta - 4\right) d\theta$$

$$= \int_{-\pi/6}^{7\pi/6} \frac{1}{2}\left(5 + 4\overset{\frac{1}{2}(1-\cos 2\theta)}{\sin^2\theta} + 12\sin\theta\right) d\theta$$

$$= \frac{1}{2}\int_{-\pi/6}^{7\pi/6} 7 - 2\cos 2\theta + 12\sin\theta \; d\theta$$

$$= \frac{1}{2}\left[7\theta - \sin(2\theta) - 12\cos\theta\right]_{-\pi/6}^{7\pi/6} = 24.187$$

Now, the other way around.

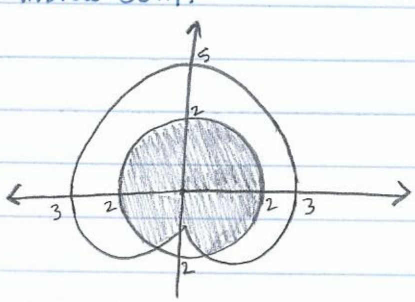


$$A = \int_{7\pi/6}^{11\pi/6} \frac{1}{2}\left((2)^2 - (3+2\sin\theta)^2\right) d\theta$$

$$= \frac{1}{2}\int_{7\pi/6}^{11\pi/6} -\left[5 + 12\sin\theta + 4\sin^2\theta\right] d\theta$$

$$= -\frac{1}{2}\left[7\theta - 12\cos\theta - \sin(2\theta)\right]_{7\pi/6}^{11\pi/6} = 2.196$$

Inside both:



$$A_{both} = A_{circle} - A_{just\text{-}found}$$

$$= \pi(2)^2 - 2.196$$

$$= 10.370$$

$$A_{both} = A_{heart} - A_1$$

$$= \int_0^{2\pi} \frac{1}{2}(3+2\sin\theta)^2 \, d\theta - 24.187$$

$$= \frac{1}{2}\int_0^{2\pi} 9 + \overset{\frac{4}{2}(1-\cos 2\theta)}{4\sin^2\theta} + 12\sin\theta \; d\theta - 24.187$$

$$= \frac{1}{2}\int_0^{2\pi} 11 - 2\cos 2\theta + 12\sin\theta \; d\theta - 24.187$$

$$= \frac{1}{2}\left[11\theta - \sin 2\theta - 12\cos\theta\right]_0^{2\pi} - 24.187$$

$$= 11\pi - 24.187$$

$$= 10.370$$

■ 2nd Exam: On Tuesday.

- Arc-length: $L = \int ds$
- Surface Area: $SA = \int 2\pi \cdot y \cdot ds$ rotated around x-axis

  $SA = \int 2\pi \cdot x \cdot ds$ rotated around y-axis

where:

$$ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx \quad \text{for } y = f(x)$$

$$ds = \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy \quad \text{for } x = f(y)$$

$$ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt \quad \text{for}$$

parametric only
$x = f(t)$
$y = f(t)$

- Inner/Outer Areas.

■ Sequences:

→ Notation: $\{a_0, a_1, a_2, \longrightarrow$ if stops $\ldots \overset{\text{general term.}}{a_n}\}$ finite.

if not, then infinite.

→ Can be given by a general formula:

$\{a_n\}_{n=0}^{\infty}$ Infinite Sequences

Examples:

- $\left\{\frac{n+1}{n^2}\right\}_{n=1}^{\infty} = \{\underset{n=1}{2}, \underset{n=2}{\frac{3}{4}}, \underset{n=3}{\frac{4}{9}}, \frac{5}{16}, \ldots$

  ↳ only naturals, not zero (in this case).

- $\left\{\frac{(-1)^{n+1}}{2^n}\right\}_{n=0}^{\infty} = \{\underset{n=0}{-1}, \underset{n=1}{\frac{1}{2}}, \underset{n=2}{\frac{-1}{4}}, \underset{n=3}{\frac{1}{8}}, \ldots\}$ This is called ALTERNATING.