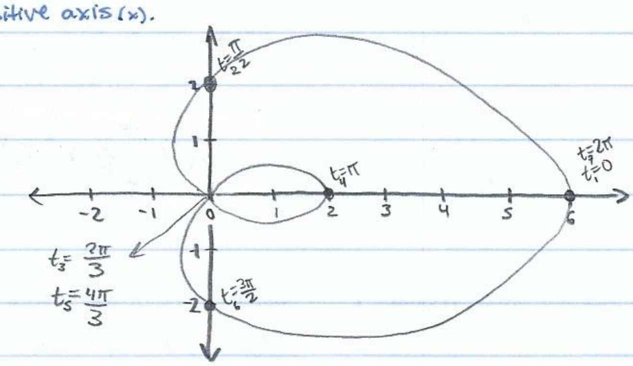
02-26-2019

Ex: Graph: $r = 2 + 4\cos\theta$

$\overbrace{a < b}^{\text{loop}}$ → symm. about x-axis

over on the positive axis (x).

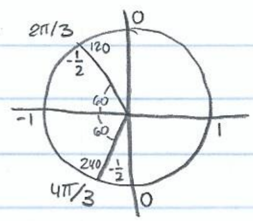
θ	r	(r,θ)
0	6	$(6,0)$
$\pi/2$	2	$(2,\frac{\pi}{2})$
π	-2	$(-2,\pi)$
$3\pi/2$	2	$(2,\frac{3\pi}{2})$
2π	6	$(6,2\pi)$



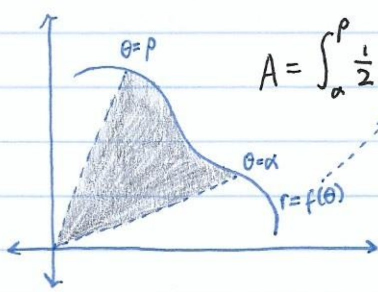
How do we know we went through the origin?
i.e. when is "r" zero? [ORIGIN]

$$r = 0 = 2 + 4\cos\theta$$

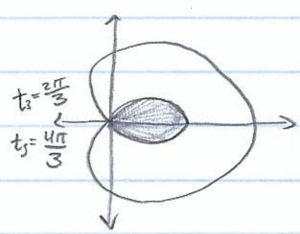
$$\frac{-1}{2} = \cos\theta \rightarrow \theta = \cos^{-1}\left(\frac{-1}{2}\right) = \underbrace{\frac{2\pi}{3}}_{\text{calc. answer}}, \frac{4\pi}{3}$$



Area with polar coordinates:



$$A = \int_{\alpha}^{\beta} \frac{1}{2} r^2 \, d\theta$$



Ex: find the area inside the inner loop of $r = 2 + 4\cos\theta$

$$A = \int \frac{1}{2} r^2 d\theta$$

$$= \int_{2\pi/3}^{4\pi/3} \frac{1}{2}(2+4\cos\theta)^2 d\theta = \frac{1}{2}\int_{2\pi/3}^{4\pi/3} 4 + 16\cos^2\theta + 16\cos\theta \, d\theta = \int_{2\pi/3}^{4\pi/3} 2 + 8\cos^2\theta + 8\cos\theta \, d\theta$$

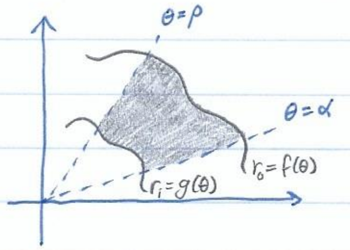
$$= 2\int_{2\pi/3}^{4\pi/3} d\theta + \cancel{8}^{4}\int_{2\pi/3}^{4\pi/3} \frac{1}{\cancel{2}}(1+\cos 2\theta)\, d\theta + 8\int_{2\pi/3}^{4\pi/3} \cos\theta \, d\theta$$

$$= 2[\theta]_{2\pi/3}^{4\pi/3} + 4\left[\theta + \frac{\sin 2\theta}{2}\right]_{2\pi/3}^{4\pi/3} + 8[\sin\theta]_{2\pi/3}^{4\pi/3}$$

$$= 4\pi - 6\sqrt{3} = 2.1740$$

Exam: Next Tuesday

■ Between curves:



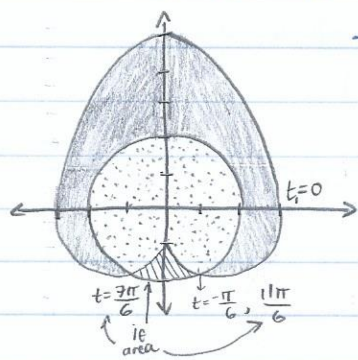
$$\text{Area}=\int_\alpha^\beta \tfrac{1}{2} r_o^2\,d\theta - \int_\alpha^\beta \tfrac{1}{2} r_i^2\,d\theta$$
$$=\int_\alpha^\beta \tfrac{1}{2}(r_o^2-r_i^2)\,d\theta$$
$$\boxed{A=\int_\alpha^\beta \tfrac{1}{2}(r_o^2-r_i^2)\,d\theta}$$

Ex: Find the area inside $r=3+2\sin\theta$ [no origin] and outside $r=2$.

circle

symm. y axis (pos. side)

θ	r
0	3
$\pi/2$	5
π	3
$3\pi/2$	1
2π	3



→ Where intersection?

$2=3+2\sin\theta$

$\frac{-1}{2}=\sin\theta$

$\theta=\frac{-\pi}{6}, \frac{7\pi}{6}$

→ What about finding all the area covered by both shapes?