**Ex: Graph:** \( r = 2 + 4 \cos \theta \)

<table>
<thead>
<tr>
<th>( \theta )</th>
<th>( r )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>( \pi/2 )</td>
<td>2</td>
</tr>
<tr>
<td>( \pi )</td>
<td>-2</td>
</tr>
<tr>
<td>( 3\pi/2 )</td>
<td>2</td>
</tr>
<tr>
<td>( 2\pi )</td>
<td>6</td>
</tr>
</tbody>
</table>

How do we know we went through the origin? i.e. when is \( r \) zero? [origin]

\[
r = 0 = 2 + 4 \cos \theta \\
-2 = 4 \cos \theta \\
\frac{-1}{2} = \cos \theta \\
\theta = \cos^{-1} \left( \frac{-1}{2} \right) = \frac{2\pi}{3}, \frac{4\pi}{3}
\]

**Area with polar coordinates:**

\[
A = \int_{\alpha}^{\beta} \frac{1}{2} r^2 \, d\theta
\]

**Ex: Find the area inside the inner loop of \( r = 2 + 4 \cos \theta \)**

\[
A = \int_{-\pi/3}^{\pi/3} \frac{1}{2} (2 + 4 \cos \theta)^2 \, d\theta = \frac{1}{2} \left[ 4 + 16 \cos^2 \theta + 16 \cos \theta \right] \, d\theta \\
= \left[ 2 + 8 \cos \theta + 8 \cos \theta \right] \, d\theta \\
= 2 \int_{-\pi/3}^{\pi/3} d\theta + \frac{4}{2} \int_{-\pi/3}^{\pi/3} (1 + \cos 2\theta) \, d\theta + 8 \int_{-\pi/3}^{\pi/3} \cos \theta \, d\theta \\
= 2 \left[ \theta \right]_{-\pi/3}^{\pi/3} + 4 \left[ \theta + \sin 2\theta \right]_{-\pi/3}^{\pi/3} + 8 \left[ \sin \theta \right]_{-\pi/3}^{\pi/3} \\
= 4 \pi - 6 \sqrt{3} = 2.1740
\]
Between curves:

\[
\text{Area} = \int_\alpha^\beta \frac{1}{2} r_0^2 \, d\theta - \int_\alpha^\beta \frac{1}{2} r_1^2 \, d\theta = \int_\alpha^\beta \frac{1}{2} (r_0^2 - r_1^2) \, d\theta
\]

\[
A = \int_\alpha^\beta \frac{1}{2} (r_0^2 - r_1^2) \, d\theta
\]

**Ex:** Find the area inside \( r = 3 + 2 \sin \theta \) [no origin] and outside \( r = 2 \).

Symmetry (pos. side)

<table>
<thead>
<tr>
<th>( \theta )</th>
<th>( r )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>( \frac{\pi}{2} )</td>
<td>5</td>
</tr>
<tr>
<td>( \pi )</td>
<td>3</td>
</tr>
<tr>
<td>( 3\frac{\pi}{2} )</td>
<td>1</td>
</tr>
<tr>
<td>2\pi</td>
<td>3</td>
</tr>
</tbody>
</table>

\( \theta = \frac{\pi}{6}, \frac{5\pi}{6} \)

\( 2 = 3 + 2 \sin \theta \)
\( \frac{1}{2} = \sin \theta \)
\( \theta = \frac{\pi}{6} \)

Where intersection?

What about finding all the area covered by both shapes?