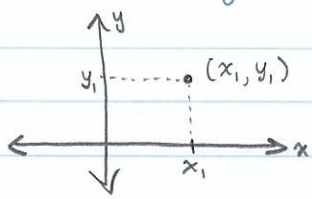
02-25-2019

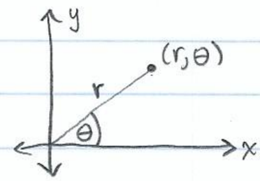
Polar Coordinates: "New set of instructions"

▶ Cartesian coord. system:



only perfectly horizontally and vertically.

▶ Polar coordinates:



Cartesian points are unique. Polar points are not unique in comparison.

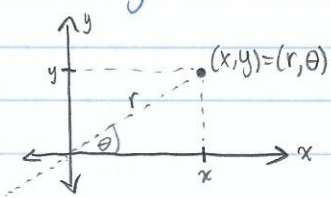
For example: $(3, \frac{\pi}{6}) = (3, \frac{\pi}{6} + 2\pi) = (3, \frac{-11\pi}{6})$

"driving in reverse" $= (-3, 7\pi/6) = (-3, -5\pi/6)$

positive dir. negative dir.

Summary: $(r, \theta) = (r, \theta \pm 2\pi n) = (-r, \theta \pm \pi n)$

▶ Converting back and forth between Polar & Cartesian:



- Polar → Cartesian:

$x = r\cos\theta$

$y = r\sin\theta$

- Cartesian → Polar:

$r = \sqrt{x^2 + y^2}$ (Assume posit.)

$\theta_1 = \tan^{-1}(\frac{y}{x})$ $\theta_2 = \theta_1 + \pi$

↳ 2 angles!

↳ $x=0$ breaks it.

→ Calculators only give answers to arctan in the 1st or 4th quadrant.

Ex:

Convert $(-1,1)$ to polar coordinates (r,θ).

$r = \sqrt{(-1)^2 + (1)^2}$ $\theta_1 = \arctan(\frac{1}{-1})$ $\theta_2 = \theta_1 + \pi = \frac{3\pi}{4}$

$r = \sqrt{2}$ $= \arctan(-1) = \frac{-\pi}{4}$ (calc)

→ Now θ_1 or θ_2? Locate $(-1,1)$ in its quadrant and decide.

It's $\theta_2 = \frac{3\pi}{4}$ since $(-1,1)$ is in the 2nd quadrant.

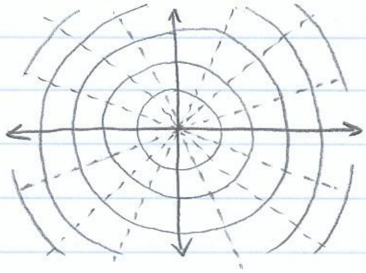
→ $-\frac{\pi}{4}$ can be used too, but r would have to be $-\sqrt{2}$.

→ Note:

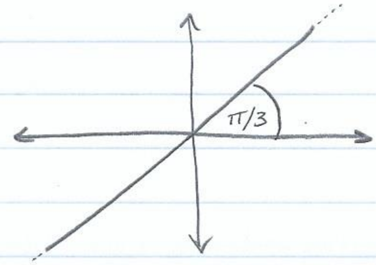
$(-1, 1) = (\sqrt{2}, \frac{3\pi}{4}) = (-\sqrt{2}, -\frac{\pi}{4})$ EQUIVALENT, NOT EQUAL.

POLAR GRAPHS:

Original polar template:

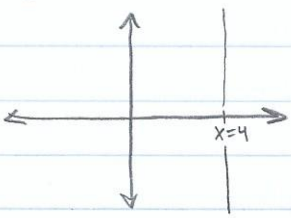


(a) $\theta = \pi/3 \rightarrow (r, \pi/3)$ where $-\infty < r < \infty$

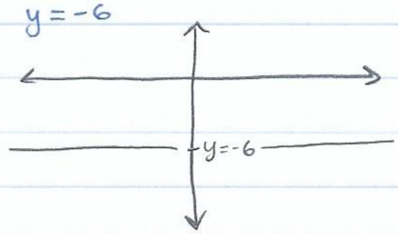


(b) $r\cos\theta = 4$
$x = 4$

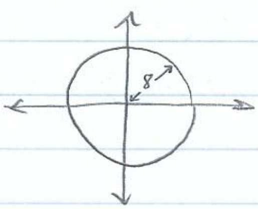
polar → cartesian.



(c) $r\sin\theta = -6$
$y = -6$



(d) $r = 8 \rightarrow (8, \theta)$ where $0 \le \theta \le 2\pi$



(e) $r = 6\cos\theta$

$r^2 = 6r\cos\theta$

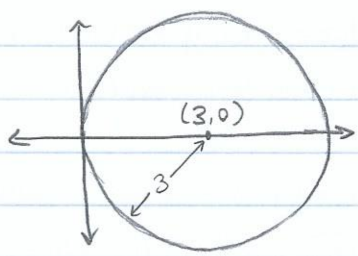
$x^2 + y^2 = 6x$

$x^2 - 6x + 3^2 - 3^2 + y^2 = 0$

$(x-3)^2 + y^2 = 9$

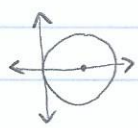
$r = 3$, center at $(3,0)$

Still a circle

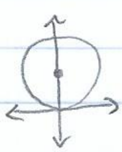


Summary:

- $r = 2a\cos\theta$
  center: $(a, 0)$
  radius: $|a|$

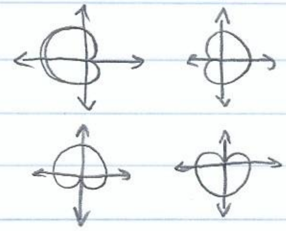


- $r = 2b\sin\theta$
  center: $(0, b)$
  radius: $|b|$



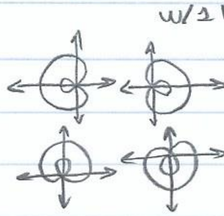
- $r = a \pm a\cos\theta$
- $r = b \pm b\sin\theta$

CARDIOIDS



- $r = a \pm b\cos\theta$
- $r = a \pm b\sin\theta$

$a < b$ limacon w/ 1 loop



- $r = a \pm b\cos\theta$
- $r = a \pm b\sin\theta$

$a > b$