02-22-2019

Ex: Find the length of:

- $x = 3\sin(t)$
- $y = 3\cos(t)$

on $0 \le t \le 2\pi$ $\qquad ds = \sqrt{\left[\frac{dx}{dt}\right]^2 + \left[\frac{dy}{dt}\right]^2}\,dt$

$$ds = \sqrt{[3\cos(t)]^2 + [-3\sin(t)]^2}\,dt$$

$$= \sqrt{9\cos^2 t + 9\sin^2 t} = \sqrt{9(\cos^2 t + \sin^2 t)} = \sqrt{9} = 3dt$$

$$L = \int ds \Rightarrow L = \int_0^{2\pi} 3dt = 3t\Big|_0^{2\pi} = 6\pi$$

Next time be careful with the limits not going over one trace.
In this problem we were lucky to be given a range for one trace exactly.

Ex: Find the length of:

$x = 3\sin(3t)$ on $\underbrace{0 \le t \le 2\pi}$
$y = 3\cos(3t)$ More than one trace! Exactly 3 traces.

$L = \int ds \qquad ds = \sqrt{[9\cos(3t)]^2 + [-9\sin(3t)]^2}\,dt$

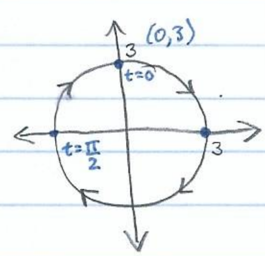
$$= \sqrt{81(\cos^2(3t) + \sin^2(3t))}\,dt = 9dt = ds$$

We know we go around the circle 3 times:

$$L = \int_0^{2\pi} 9\,dt = 18\pi$$

$2\pi \longrightarrow$ Not one trace! wrong answer!!

18π ↳ Total distance traveled.



I haven't yet obtained the right answer.

→ We need a range of t for one trace.

- Where do we start? $t=0$

  $x=0$

  $y=3$

- Direction:

  $\frac{dx}{dt} = 9\cos(3t)$ So, $\frac{dx}{dt} > 0$ for

  $0 \le 3t \le \pi/2$

  pos. $0 \le t \le \pi/6$

  Then we're moving to the right when we're at $(0,3)(t=0)$.

- Where's for $t = \pi/2$?

  $x = -3$ cw? ccw?

  $y = 0$

- The argument always goes back to starting pt in the $0, 2\pi$ range:

$$0 \le 3t \le 2\pi$$

$$0 \le t \le \frac{2\pi}{3} \text{ range for one trace.}$$

Thus, we now can compute the arclength:

$$L=\int ds$$

$$L=\int_0^{2\pi/3} 9\,ds = 9t\Big|_0^{2\pi/3} = 9\left(\frac{2\pi}{3}\right) = 6\pi.$$

Surface area w/ parametric equations:

$$x = f(t) \quad \text{for } \underbrace{a \le t \le b}_{\text{exactly one trace}}.$$

$$y = h(t)$$

<u>Ex</u>: Rotate about the x-axis.

$$x = \cos^3\theta \quad \text{for } 0 \le \theta \le \pi/2$$

$$y = \sin^3\theta \qquad \text{one trace}$$

$$SA = \int 2\pi y\, ds \quad , \quad ds = \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2}\, d\theta$$

$$= \sqrt{[-3\cos^2\theta\sin\theta]^2 + [3\sin^2\theta\cos\theta]^2}\, d\theta$$

$$= \sqrt{9\cos^4\theta\sin^2\theta + 9\sin^4\theta\cos^2\theta}\, d\theta$$

$$= \sqrt{9\cos^2\theta\sin^2\theta(\cos^2\theta + \sin^2\theta)}\, d\theta$$

$$= 3|\cos\theta\sin\theta|\, d\theta$$ because of the range, $\sin\theta$ & $\cos\theta$ are both always positive, so we can drop absval.

$$= 3\cos\theta\sin\theta\, d\theta$$

$$SA = \int_0^{\pi/2} 2\pi(\sin^3\theta)\, 3\cos\theta\sin\theta\, d\theta$$

$$= 6\pi\int_0^{\pi/2} \sin^4\theta\cos\theta\, d\theta \qquad u=\sin\theta,\ du=\cos\theta\, d\theta$$

$$= 6\pi\int_0^{\pi/2} u^4\, du \qquad u=\sin(\tfrac{\pi}{2})=1,\ u=\sin(0)=0$$

$$= 6\pi\int_0^1 u^4\, du = 6\pi\left[\frac{u^5}{5}\right]_0^1 = 6\pi\left(\frac{1}{5}\right) = \frac{6\pi}{5}$$