Ex: Find the length of:

\[ x = 3 \sin(t) \quad \text{on} \quad 0 \leq t \leq 2\pi \quad ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \, dt \]
\[ y = 3 \cos(t) \]

\[ ds = \sqrt{[3 \cos(t)]^2 + [-3 \sin(t)]^2} \, dt \]
\[ = \sqrt{9 \cos^2 t + 9 \sin^2 t} = \sqrt{9(\cos^2 t + \sin^2 t)} = \sqrt{9} = 3 \, dt \]

\[ L = \int ds \Rightarrow L = \int_0^{2\pi} 3 \, dt = 3t \bigg|_0^{2\pi} = 6\pi \]

Next time be careful with the limits not going over one trace.
In this problem we were lucky to be given a range for one trace exactly.

Ex: Find the length of:

\[ x = 3 \sin(3t) \quad \text{on} \quad 0 \leq t \leq 2\pi \]
\[ y = 3 \cos(3t) \quad \text{More than one trace! Exactly 3 traces.} \]

\[ L = \int ds \]
\[ ds = \sqrt{[9 \cos(3t)]^2 + [-9 \sin(3t)]^2} \, dt \]
\[ = \sqrt{81 \cos^2(3t) + 81 \sin^2(3t)} \, dt = 9 \, dt = ds \]

We know we go around the circle 3 times:

\[ L = \int_0^{2\pi} 9 \, dt = 18\pi \]
\[ \Rightarrow \text{Total distance traveled.} \]

I haven't yet obtained the right answer.
- We need a range at t for one trace.
  - Where do we start? \( t = 0 \)
  - Direction:
    \[ \frac{dx}{dt} = 9 \cos(3t) \quad \text{at} \quad 0 \leq t \leq \frac{\pi}{6} \]
    \[ \frac{dy}{dt} = -9 \sin(3t) \quad \text{for} \quad 0 \leq t \leq \frac{\pi}{6} \]
  - Where for \( t = \frac{\pi}{2} \)? Then we were moving to the right
    \[ y = -3 \quad \text{cw? ccw?} \quad \text{When we're at (0,3) (t = 0)}. \]
    \[ y = 0 \]
  - The argument always goes back to starting pt in the \( 0, 2\pi \) range:
    \[ 0 \leq 3t \leq 2\pi \]
    \[ 0 \leq t \leq \frac{2\pi}{3} \] range for one trace.
Thus, we now can compute the arc length:

\[
L = \int ds = \int_0^{2\pi/3} 9t\ dt = 9\left(\frac{2\pi}{3}\right) = 6\pi.
\]

**Surface area w/ parametric equations:**

\[
\begin{align*}
& \text{X = } f(t) \quad \text{for } a \leq t \leq b. \\
& \text{Y = } g(t) \quad \text{exactly one trace.}
\end{align*}
\]

**Ex:** Rotate about the y-axis.

\[
\begin{align*}
& X = \cos^3\theta \quad \text{for } 0 \leq \theta \leq \pi/2 \\
& Y = \sin^3\theta \quad \text{one trace}
\end{align*}
\]

\[
\text{SA} = \int 2\pi y\ ds, \quad ds = \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2} d\theta
\]

\[
\begin{align*}
& = \sqrt{[-3\cos^2\theta\sin\theta]^2 + [3\sin^2\theta\cos\theta]^2} \ d\theta \\
& = \sqrt{9\cos^4\theta\sin^2\theta + 9\sin^4\theta\cos^2\theta} \ d\theta \\
& = 3\sqrt{\cos^2\theta\sin^2\theta} \ d\theta \\
& = 3\cos\theta\sin\theta \ d\theta \quad \text{because the range, } \sin\theta \cos\theta \text{ are both always positive, so we can drop absolute.}
\end{align*}
\]

\[
\text{SA} = \int_0^{\pi/2} 2\pi (\sin^3\theta) 3 \cos^2\sin\theta \ d\theta
\]

\[
\begin{align*}
& = 6\pi \int_0^{\pi/2} \sin^4\theta \cos\theta \ d\theta \\
& = 6\pi \int_0^{\pi/2} u^4 \ du, \quad u = \sin\left(\frac{\theta}{2}\right)
\end{align*}
\]

\[
\begin{align*}
& = 6\pi \left[ \frac{u^5}{5} \right]_0^{\pi/2} = 6\pi \left(\frac{1}{5}\right) = \frac{6\pi}{5}.
\end{align*}
\]