Ex: Find the tangent line(s) to: at \((0,4)\)

\[ x = t^5 - 4t^3 \]
\[ y = t^2 \]

- Slope: \( m = \frac{dy}{dx} \). We're going to do it without eliminating the parameter.

Remember:

\[ \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2t}{5t^4 - 12t^2} = \frac{2}{5t^2 - 12t} \]

\[ M = \frac{dy}{dx} \bigg|_{t=0} = \frac{2}{t=0} \]

\[ \frac{dy}{dx} = \frac{2}{t=0} \]

\[ M = \frac{dy}{dt} \bigg|_{t=0} = \frac{2}{t=0} \]

\[ y = t^2 \]

\[ y_1 = 4 - \frac{1}{8}(x - 0) = 4 - \frac{1}{8}x \]

- \( t = 2 \):

\[ m = \frac{dy}{dx} \bigg|_{t=2} = \frac{1}{8} \]

\[ y_2 = 4 + \frac{1}{8}(x) = 4 + \frac{1}{8}x \]

- Where's increasing or decreasing?

\[ \frac{dy}{dx} > 0 \text{ or } \frac{dy}{dx} < 0 \]

\[ \frac{dy}{dx} = \frac{0}{5t^2 - 12t} \]

\[ > 0 \text{ or } < 0 \]

\[ \frac{dy}{dx} \]

Geometric interpretation:

Concavity: second derivative. \( \frac{d^2y}{dx^2} \)

\[ \frac{d^2y}{dx^2} = \frac{d(dy/dx)}{dx} \]

Look at the derivative as a function.

\[ \Rightarrow \frac{d}{dx}(y) = \frac{d}{dx}(t^5 - 4t^3) \]

This is my new function.

\[ x = t^5 - 4t^3 \]

\[ y = t^2 \]

\[ \frac{d}{dx}(t^3) = \frac{d}{dx}(t^5 - 4t^3) = \frac{2t}{5t^4 - 12t^2} \]

\[ \big( \frac{d}{dx}(t^2) \big) = \frac{2t}{5t^4 - 12t^2} \]

\[ \frac{d}{dx}(t^2) = \frac{d}{dx}(t^5 - 4t^3) \]

\[ \frac{d^2y}{dx^2} = \frac{d}{dx} \left( \frac{dy}{dx} \right) \]

Then...

\[ \frac{d^2y}{dx^2} = \frac{d}{dx} \left( \frac{dy}{dx} \right) \]
Example:

\[ x = t^5 - 4t^3 \]
\[ y = t^2 \]

\[ \frac{dx}{dt} = \frac{2}{5t^3} \quad \frac{d}{dt} \left( \frac{dy}{dx} \right) = \frac{d}{dt} \left( \frac{2}{5t^3 - 12t} \right) \]

\[ \frac{dx}{dt} = 5t^4 - 12t^2 \]

\[ \frac{d^2y}{dx^2} = \frac{-2 \left( 15t^2 - 12 \right) \left( 5t^3 - 12t \right)^2}{5t^4 - 12t^2} \]

**Arc Length (parametric equations):**

Given: \( x = f(t) \) on \( a \leq t \leq b \)

\[ L = \int_{a}^{b} ds \]

What will it be this time?

- \( t \)-range for one trace.

If \( ds = \sqrt{1 + \left( \frac{dx}{dt} \right)^2} \), \( dx \) will need \( y = f(x) \)

If \( ds = \sqrt{1 + \left( \frac{dy}{dt} \right)^2} \), \( dy \) will need \( x = f(y) \)

If \( ds = \sqrt{\left( \frac{dx}{dt} \right)^2 + \left( \frac{dy}{dt} \right)^2} \), \( dt \) will need \( x = f(t) \)

If \( ds = \sqrt{\left( \frac{dx}{dt} \right)^2 + \left( \frac{dy}{dx} \right)^2} \), \( dt \) will need \( y = h(t) \)