02-21-2019

Ex. Find the tangent line(s) to: at $(0,4)$

$$x = t^5 - 4t^3$$
$$y = t^2$$

▸ Slope: $m = \frac{dy}{dx}$. We're going to do it without eliminating the parameter.

Remember:

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2t}{5t^4 - 12t^2} = \frac{2}{5t^3 - 12t}$$

$m = \left.\frac{dy}{dx}\right|_{\substack{x=0 \\ y=4}}$ what t?

$0 = t^5 - 4t^3$ $\quad$ $4 = t^2$

$4t^3 = t^5$ $\quad$ $t = \pm 2$

$4 = t^2$

$t = \pm 2, 0$

Which work? 0 doesn't. ± 2 do.

- $t = -2$:

$$m = \left.\frac{dy}{dx}\right|_{t=-2} = -\frac{1}{8}$$

tangent line: $(0,4)$

$$y_1 = 4 - \frac{1}{8}(x - 0) = 4 - \frac{1}{8}x$$

- $t = 2$:

$$m = \left.\frac{dy}{dx}\right|_{t=2} = \frac{1}{8}$$

tangent line: $(0,4)$

$$y_2 = 4 + \frac{1}{8}(x) = 4 + \frac{1}{8}x$$

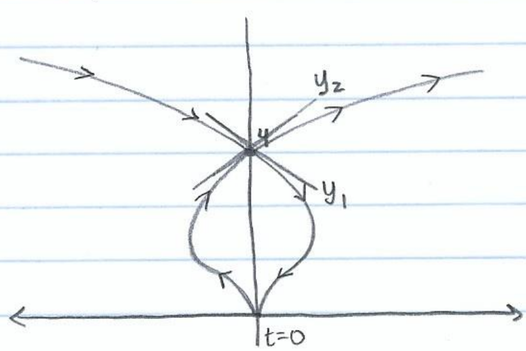
→ Where's increasing or decreasing?

$\frac{dy}{dx} > 0$ or $\frac{dy}{dx} < 0$

So we'd need to do:

$$\frac{dy}{dx} = \frac{2}{5t^3 - 12t} > 0 \text{ or } < 0$$

Geometric Interpretation:



Concavity: second derivative. d^2y/dx^2

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$ Look at the derivative as a function.

↳ $\frac{d}{dx}(y) = \frac{\frac{d}{dt}(y)}{dx/dt}$ this is my new function. $\quad x = t^5 - 4t^3$, $y = t^2$

$$\frac{d}{dx}(t^2) = \frac{\frac{d}{dt}(t^2)}{5t^4 - 12t^2} = \underbrace{\frac{2t}{5t^4 - 12t^2}}_{f(t)}$$, Then...

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\underbrace{\left(\frac{dy}{dx}\right)}_{f(t)} = \frac{\frac{d}{dt}(f(t))}{dx/dt} \longrightarrow \boxed{\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{dx/dt}}$$

Ex: $x = t^5 - 4t^3$

$y = t^2$

• $\frac{dy}{dx} = \frac{2}{5t^3-12t}$, $\frac{d}{dt}\left(\frac{dy}{dx}\right) = \frac{d}{dt}\left(\frac{2}{5t^3-12t}\right)$

$= \frac{-2}{(5t^3-12t)^2}(15t^2-12)$

• $\frac{dx}{dt} = 5t^4 - 12t^2$

$$\frac{d^2y}{dx^2} = \frac{-2(15t^2-12)(5t^3-12t)^{-2}}{5t^4-12t^2}$$

ARC - LENGTH (parametric equations):

Given: $x = f(t)$ on $\underline{a \le t \le b}$
$y = h(t)$ t-range for one trace.

$L = \int ds$ what will it be this time?

if $ds = \sqrt{1 + (dy/dx)^2}\,dx$ will need $y = f(x)$

if $ds = \sqrt{1 + (dx/dy)^2}\,dy$ will need $x = f(y)$

if $ds = \sqrt{\left[\frac{dx}{dt}\right]^2 + \left[\frac{dy}{dt}\right]^2}\,dt$ will need $x = f(t)$, $y = h(t)$