02-20-2019

HW help session:

· **Ex** $x=\sqrt{2+4\sin^2(3t)} \rightarrow x^2=2+4\sin^2(3t)$

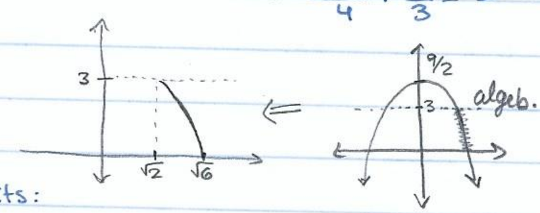
$y=3\cos^2(3t)$ $\qquad \frac{x^2-2}{4}=\sin^2(3t)$

$\frac{y}{3}=\cos^2(3t)$ since, $\sin^2 3t+\cos^2 3t=1$

then, $\frac{x^2-2}{4}+\frac{y}{3}=1 \rightarrow \frac{y}{3}=1-\frac{x^2-2}{4}$

$y=3-\frac{3x^2-6}{4}=\frac{-3}{4}x^2+3+\frac{3}{2}$

$y=-\frac{3}{4}x^2+\frac{9}{2}$



→ Limits:

$0\le\cos^2(3t)\le 1$

$0\le 3\cos^2(3t)\le 3$

$0\le y\le 3$

Now, looks like we have two paths, but we should only have 1. Since $x>0$, we have it.

$0\le\sin^2(3t)\le 1$

$0\le 4\sin^2(3t)\le 4$

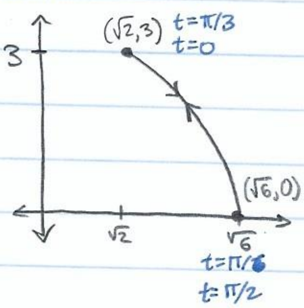
$2\le 4\sin^2(3t)+2\le 6$

$\sqrt{2}\le\sqrt{4\sin^2(3t)+2}\le\sqrt{6}$

$\sqrt{2}\le x\le\sqrt{6}$

→ Direction: it'll oscillate cause of the sine, cosine on x and y.

→ Trace:



$y=0$: $3\cos^2(3t)=0$

$\cos^2(3t)=0$

$\cos(3t)=0$

$3t=\frac{\pi}{2}$ or $\frac{3\pi}{2}$

$3t=\frac{3\pi}{2}+2\pi n$ $\quad 3t=\frac{\pi}{2}+2\pi n$

$t=\frac{\pi}{2}+\frac{2\pi}{3}n$ $\quad t=\frac{\pi}{6}+\frac{2\pi}{3}n$

$y=3$: $\cos^2(3t)=1$

$\cos(3t)=\pm 1$

$\cos(3t)=1$ $\qquad \cos(3t)=-1$

$3t=0+2\pi n$ $\qquad 3t=\pi+2\pi n$

$t=\frac{2\pi}{3}n$ $\qquad t=\frac{\pi}{3}+\frac{2\pi}{3}n$

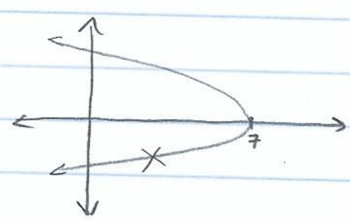
Range: $0\le t\le\frac{\pi}{6}$; $\pi/6\le t\le\pi/3$; $\pi/3\le t\le\pi/2$

· **Ex:** $x=7-36e^{-4t}$ $\quad$ Notice $e^{-4t}=(e^{-2t})^2$

$y=3e^{-2t}$ $\quad x=7-36(e^{-2t})^2$

$\frac{y}{3}=e^{-2t}$ $\quad x=7-36\left(\frac{y}{3}\right)^2$

$x=7-4y^2$

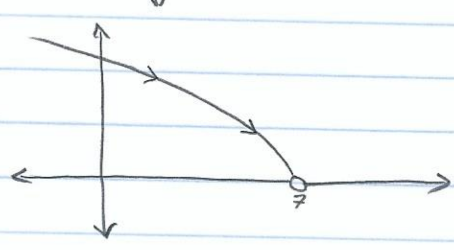


Remember $y=3e^{-2t}>0$

$e^{-4t}>0$

$-36e^{-4t}<0$

$x=7-36e^{-4t}<7$

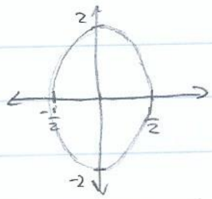


Direction: $\frac{dy}{dt}=-6e^{-2t}$ → y decreases

This does one trace only $(-\infty,\infty)$

Ex: #8 (Set up only)

$4x^2 + \frac{y^2}{4} = 1$



$y^2 = 4 - 16x^2$

$y = \pm\sqrt{4-16x^2}$

$y = +\sqrt{4-16x^2}$ top half, then $\times 2$

↳ $\frac{dy}{dx}$ → $ds = \sqrt{1 + (dy/dx)^2}\, dx$

$$L = 2\int_{-1/2}^{1/2} \sqrt{\quad}\; dx$$

→ If we want to rotate around x-axis:

$$SA = \int_{-1/2}^{1/2} 2\pi y \, ds = 2\pi \int_{-1/2}^{1/2} \sqrt{4-16x^2}\,\sqrt{\cdots}\; dx$$