02-19-2019

Ex:

$x = 3\cos(2t)$, $y = 1 + \cos^2(2t)$

$\boxed{1}$ Sketch curve

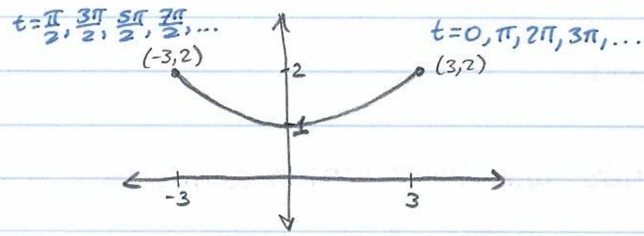
$\boxed{2}$ Limits on x and y

$\boxed{3}$ Range for one trace

$\boxed{4}$ Number of traces (on $-\pi \le t \le 50\pi$)

So, we found...

$$y = 1 + \frac{x^2}{9} \qquad -3 \le x \le 3 \quad \text{and} \quad 1 \le y \le 2$$



How to find how long a trace takes?

Another way of asking this is which t's leave me on $(3,2)$? $(-3,2)$

$x=3$: $\quad 3 = 3\cos(2t) \Rightarrow \cos(2t) = 1 \rightarrow 2t = 0 + 2\pi k \quad k = 0,1,2,\ldots$

$x=-3$: $\quad -3 = 3\cos(2t) \Rightarrow \cos(2t) = -1 \rightarrow 2t = \pi + 2\pi k \quad k = 0,1,2\ldots$

$t = \pi k \qquad t = \frac{\pi}{2} + \pi k \quad k = 0,1,2,\ldots$

So, range of t's for one range:

$\underbrace{0 \le t \le \pi/2}_{\text{easier to deal with...}} \qquad \pi/2 \le t \le \pi \qquad \pi \le t \le \frac{3\pi}{2}$ (pick any).

- # of traces:

$$\frac{\text{Total time traveled}}{\text{time of a trace}} = \frac{50\pi - (-\pi)}{\pi/2} = \frac{51\pi}{\pi/2} = 51\cancel{\pi} \times \frac{2}{\cancel{\pi}} = 102 \text{ traces!}$$

Parametrize a Curve:

Ellipse: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

one way:

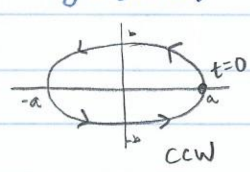
$x = a\cos(t)$
$y = b\sin(t)$

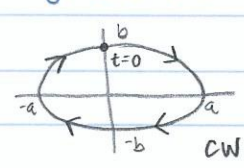
or

$x = a\sin(t)$
$y = b\cos(t)$

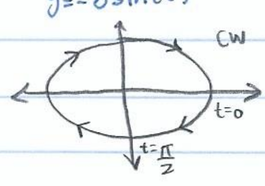
$x' = a\cos t \qquad 0 \le t \le \pi/2$
$y' = -b\sin t \qquad x' > 0$
$y' < 0$

What about direction?





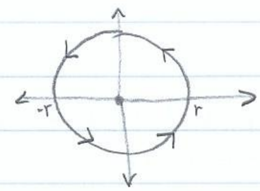
$x = a\cos(t)$
$y = -b\sin(t)$



circle: $x^2+y^2=r^2$

$x = r\cos(t)$

$y = r\sin(t)$



ex: $y=f(x)$

Let's say $y = x^2-3x+1$

$x = t$

$y = t^2-3t+1$

or more generally $x=t$, $y=f(t)$

TANGENT LINES with PARAMETRIC EQUATIONS:

So, given:

$x = f(t)$

$y = h(t)$

review: $y=F(x)$ at $x=a$

point: $(a, F(a))$

slope: $m = F'(a) = \left.\dfrac{dy}{dx}\right|_{x=a}$

Let's suppose we can eliminate the parameter. We get:

$y=F(x)$ But this won't be easily done most of the time.

Put the parameter back in.

$h(t) = F(f(t))$

Let's differentiate with respect to t:

$$h'(t) = \underbrace{F'(f(t))}_{\text{with respect to } x} \cdot \underbrace{f'(t)}_{\text{with respect to } t} \rightarrow F'(f(t)) = \frac{h'(t)}{f'(t)}$$

Thus, $\dfrac{dy}{dt} = \dfrac{dy}{dx}\cdot\dfrac{dx}{dt} \longrightarrow \boxed{\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt}, \text{ (hope) } \dfrac{dx}{dt} \neq 0}$

Notice too:

$$\boxed{\frac{dx}{dy} = \frac{dx/dt}{dy/dt}, \text{ (hope) } \frac{dy}{dt} \neq 0}$$