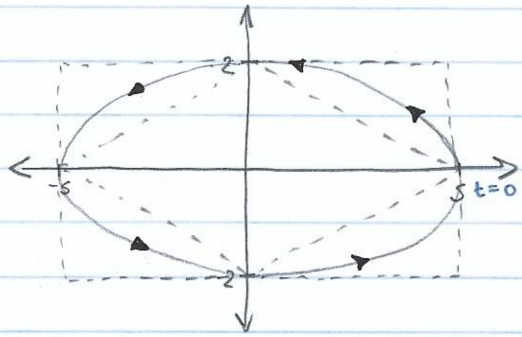
02-18-2019

**Ex:** $x = 5\cos(t)$ for $0 \le t \le 2\pi$

$y = 2\sin(t)$

| $t$ | $x$ | $y$ |
|---|---|---|
| $0$ | $5$ | $0$ |
| $\frac{\pi}{2}$ | $0$ | $2$ |
| $\pi$ | $-5$ | $0$ |
| $\frac{3\pi}{2}$ | $0$ | $-2$ |
| $2\pi$ | $5$ | $0$ |




¿?

→ Eliminate parameter:

- $t = \cos^{-1}\left(\frac{x}{5}\right) \rightarrow y = 2\sin\left(\cos^{-1}\left(\frac{x}{5}\right)\right)$ ??? Not easy to deal with, at all!
- $\cos(t) = \frac{x}{5}$, $\sin(t) = \frac{y}{2}$, we know $\cos^2(t) + \sin^2(t) = 1$

$$\frac{x^2}{25} + \frac{y^2}{4} = 1 \Rightarrow \text{ellipse!}$$

Now we clearly see what this is.

$$\left(\frac{x}{5}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$$

Reminder: we should not have assumed we have the entire ellipse.

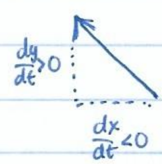
→ Directions:

$\frac{dx}{dt} = -5\sin(t)$ $\quad \frac{dy}{dt} = 2\cos(t)$

Look at the range: $0 \le t \le \pi/2$

$\frac{dx}{dt} < 0$, $\frac{dy}{dt} > 0$

alone is useless




on $\left[0, \frac{\pi}{2}\right]$ So, counterclockwise.

As long as cos, sin are raised only to the 1st power (same for t), whichever direction we have on the 1st quadrant, it'll be kept (continued) in the next quadrants. No need to worry about change of behavior.

Ex: $x = 5\cos(3t)$ for $0 \le t \le 2\pi$

$y = 2\sin(3t)$

| $t$ | $x$ | $y$ |
|---|---|---|
| 0 | 5 | 0 |
| $\pi/2$ | 0 | -2 |
| $\pi$ | -5 | 0 |
| $3\pi/2$ | 0 | 2 |
| $2\pi$ | 5 | 0 |

- Looks like the graph of this will be an ellipse again.
- By looking at the points it seems like we're going clockwise!

▸ Direction: $\frac{dx}{dt} = -15\sin(3t)$ $\frac{dy}{dt} = 6\cos(3t)$

Different range: $0 \le t \le \frac{\pi}{6}$ $\frac{dx}{dt} < 0$, $\frac{dy}{dt} > 0$

So that we stay on the first quadrant.

→ Easier way: How many times are going around the ellipse?

First answer, when do we complete our first revolution?

$(5,0) \rightarrow 5 = 5\cos(3t)$ & $0 = 2\sin(3t)$

[true at 1 location] $1 = \cos(3t)$ & $0 = \sin(3t)$ [true at 2 locations]

$3t = 0 + 2\pi n$ where $n = 0, 1, 2, \ldots$

$3t = 2\pi n \rightarrow t = \frac{2}{3}\pi n$

Then we're at the origin at:

$$t = 0, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{6\pi}{3} = 2\pi$$

1 rev, 2, 3 — Then, 3 revolutions!

Ex: $x = 3\cos(2t)$

$y = 1 + \cos^2(2t)$

① sketch path (algebraic eqt'n only)

② limits on $x$ and $y$.

③ range of $t$'s for one trace (no portion retraced).

$\cos(2t) = \frac{x}{3}$

$y = 1 + \left(\frac{x}{3}\right)^2$

$y = 1 + \frac{x^2}{9}$

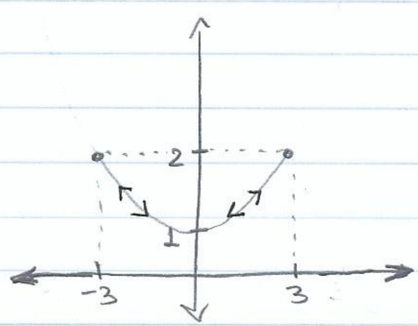
This won't be a complete parabola though.

We know: $-1 \le \cos(2t) \le 1 \Rightarrow -3 \le 3\cos(2t) \le 3$

$-3 \le x \le 3$

$0 \le \cos^2(2t) \le 1 \Rightarrow 1 \le 1 + \cos^2(2t) \le 2$

$1 \le y \le 2$




So, now we have our parametric curve (no direct. yet).