Example: \( x = 5 \cos(t) \) for \( 0 \leq t \leq 2\pi \)
\( y = 2 \sin(t) \)

<table>
<thead>
<tr>
<th>( t )</th>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>( \frac{\pi}{2} )</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>( \pi )</td>
<td>-5</td>
<td>0</td>
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<tr>
<td>( \frac{3\pi}{2} )</td>
<td>0</td>
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→ Eliminate parameter:

- \( t = \cos^{-1}\left(\frac{x}{5}\right) \rightarrow y = 2 \sin\left(\cos^{-1}\left(\frac{x}{5}\right)\right) \) \( \left(??? \text{ not easy to deal with, at all!}\right) \)
- \( \cos(t) = \frac{x}{5}, \sin(t) = \frac{y}{2} \), we know \( \cos^2(t) + \sin^2(t) = 1 \)

\( \frac{x^2}{25} + \frac{y^2}{4} = 1 \Rightarrow \text{ellipse!} \)

Now we clearly see what this is.

\( \left(\frac{x}{5}\right)^2 + \left(\frac{y}{2}\right)^2 = 1 \)

Reminder: we should not have assumed we have the entire ellipse.

→ Directions:

\( \frac{dx}{dt} = -5 \sin(t) \quad \frac{dy}{dt} = 2 \cos(t) \)

Look at the tangents: \( 0 \leq t \leq \frac{\pi}{2} \)

\( \frac{dx}{dt} < 0, \quad \frac{dy}{dt} > 0 \)

So, \( \frac{dx}{dt} < 0 \), \( \frac{dy}{dt} > 0 \)

As long as \( \cos, \sin \) are raised only to the 1st power (same for \( t \)), whenever direction we have on the 1st quadrant, it'll be kept (continued) in the next quadrants. No need to worry about change of behavior.
Ex: \( x = 5 \cos (3t) \) for \( 0 \leq t \leq 2\pi \)

\[ y = 2 \sin (3t) \]

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- Looks like the graph.
- By looking at the points it seems like we're going clockwise!

\[ \Delta x = -15 \sin (3t) \quad \Delta y = 6 \cos (3t) \]

Different range: \( 0 \leq t \leq \frac{\pi}{6} \)

\( \Delta x < 0, \quad \Delta y > 0 \)

So that we stay on the first quadrant.

→ Easier way: How many times are going around the ellipse?

First answer, when do we complete our first revolution?

\( (5, 0) \) → \( 5 = 5 \cos (3t) \) & \( 0 = 2 \sin (3t) \)

[treat 1 location] \( 1 = \cos (3t) \) & \( 0 = 2 \sin (3t) \) [treat 2 locations]

\( 3t = 0 + 2\pi n \) where \( n = 0, 1, 2, ... \)

\( 3t = 2\pi n \) → \( t = \frac{2\pi n}{3} \)

Then we're at the origin at:

\( t = 0, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{6\pi}{3} = 2\pi \)

Then, 3 revolutions!

Ex: \( x = 3 \cos (2t) \)

- Sketch, plots (algebraic sketch only)
- Limits on \( x \) and \( y \)
- Range of \( t \) for one trace (no portion retracted).

\( y = 1 + \cos^2 (2t) \)

This isn't a complete parabola, though.

\( y = 1 + \frac{x^2}{9} \)

We know:

\( -1 \leq \cos (2t) \leq 1 \) \( \Rightarrow -3 \leq 3 \cos (2t) \leq 3 \)

\( -3 \leq x \leq 3 \)

\( 0 \leq \cos^2 (2t) \leq 1 \) \( \Rightarrow 1 \leq 1 \cos^2 (2t) \leq 2 \)

\( 1 \leq y \leq 2 \)

So, now we have our parametric curve (no direct yet).