02-13-2019

Ex: Find the surface area of the solid obtained by rotating $y=\sqrt{9-x^2}$ on $-2 \le x \le 2$ about the x-axis.

$$SA = \int 2\pi y \, ds$$

$$ds = \sqrt{1+(dy/dx)^2}\,dx$$
$$= \sqrt{1+\left(-x(9-x^2)^{-1/2}\right)^2}\,dx$$
$$= \sqrt{1+x^2(9-x^2)^{-1}}\,dx$$
$$= \sqrt{1+\frac{x^2}{9-x^2}}\,dx$$
$$ds = \sqrt{\frac{9}{9-x^2}}\,dx = \underbrace{\frac{3}{\sqrt{9-x^2}}\,dx}_{\text{put in S.A.}}$$

$$SA = \int_{-2}^{2} 2\pi \cancel{\sqrt{9-x^2}} \cdot \frac{3}{\cancel{\sqrt{9-x^2}}}\,dx$$
$$= 2\pi \cdot 3 \int_{-2}^{2} dx$$
$$= 6\pi[2+2] = 24\pi$$

Remember, the answers (surfaces) should always be positive.

Ex: Rotate $y=\sqrt[3]{x}$, $1 \le y \le 2$ about the y-axis.

- Well use both ds's.

$$SA = \int 2\pi \overset{\text{about } y\text{-axis}}{x}\, ds$$

$\boxed{1}$ $$ds = \sqrt{1+(dy/dx)^2}\,dx$$
$$ds = \sqrt{1+\left(\frac{1}{3}x^{-2/3}\right)^2}\,dx$$
$$= \sqrt{1+\frac{1}{9}x^{-4/3}}\,dx$$
$$= \sqrt{1+\frac{1}{9x^{4/3}}}\,dx$$
$$ds = \sqrt{\frac{9x^{4/3}+1}{9x^{4/3}}} = \frac{\sqrt{9x^{4/3}+1}}{3x^{2/3}}\,dx$$

limits in x:

$1 = x^{1/3} \Rightarrow \underline{x=1}$; $2 = x^{1/3} \Rightarrow \underline{x = 2^3 = 8}$

$$SA = \int 2\pi x \cdot \frac{\sqrt{9x^{4/3}+1}}{3x^{2/3}}\,dx$$
$$= \frac{2\pi}{3}\int_1^8 x^{1/3}\sqrt{9x^{4/3}+1}\,dx$$

$u = 9x^{4/3}+1$

$du = \overset{3}{\cancel{9}} \cdot \frac{4}{\cancel{3}} x^{1/3}\,dx$

$du = 12x^{1/3}\,dx$

$dx = \frac{du}{12x^{1/3}}$

transform int. limits

$x=1 \to u = 9(1)^{4/3}+1 = 10$

$x=8 \to u = 9(8)^{4/3}+1 = 145$

$$= \frac{\cancel{2}\pi}{3}\int_1^8 \cancel{x^{1/3}}\, u^{1/2}\,\frac{du}{\underset{6}{\cancel{12}}\cancel{x^{1/3}}}$$
$$= \frac{\pi}{18}\int_{10}^{145} u^{1/2}\,du$$
$$= \frac{\pi}{27}\left[145^{3/2} - 10^{3/2}\right]$$
$$= 199.48$$

$\boxed{2}$ $ds = \sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$

$x = y^3$ on $1 \le y \le 2$

$ds = \sqrt{1+(3y^2)^2}\,dy$

$ds = \sqrt{1+9y^4}\,dy$

$$SA = \int 2\pi x\,ds$$

$$= \int_1^2 2\pi\, y^3\sqrt{1+9y^4}\,dy$$

$$= 2\pi\int_1^2 y^3\sqrt{1+9y^4}\,dy$$

$$= 2\pi\int_1^2 y^3\sqrt{u}\,\frac{du}{36y^3}$$

$$= \frac{\pi}{18}\int_1^2 u^{1/2}\,du$$

$$= \frac{\pi}{18}\int_{10}^{145} u^{1/2}\,du$$

$$= \frac{\pi}{9\,\cancel{18}}\left(\frac{\cancel{2}}{3}\right)\left[145^{3/2} - 10^{3/2}\right]$$

$$= \frac{\pi}{27}\left(145^{3/2} - 10^{3/2}\right)$$

$$= 199.48$$

$y=1$ → $u = 1+9(1)^4 = 10$

$y=2$ → $u = 1+9(2)^4 = 145$

$u = 1+9y^4$

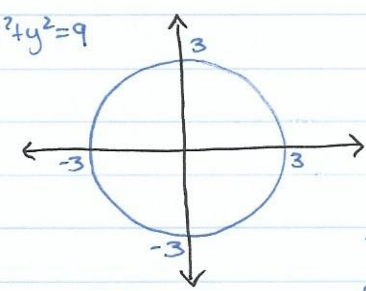
$du = 36y^3\,dy$

$\frac{du}{36y^3} = dy$

Same answer!

PARAMETRIC EQUATIONS:

$x^2+y^2=9$



A set of parametric eqtn's:

$x = f(t)$, $y = h(t)$

"t" is called the parameter

$(x,y) = (f(t), h(t))$ as t runs over some range

Think of these coordinates as the position of an object as a fxn of time.