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Ex:

- $\int \frac{x^2-29x+5}{(x-4)^2(x^2+5)}dx = \int \frac{1}{x-4} - \frac{5}{(x-4)^2} + \frac{-x+2}{x^2+3}\,dx$

$= \int \frac{1}{x-4} - \frac{5}{(x-4)^2} - \frac{x}{x^2+3} + \frac{2}{x^2+3}\,dx$

$= \ln|x-4| + \frac{5}{x-4} - \frac{1}{2}\ln|x^2+3| + \frac{2}{\sqrt{3}}\arctan\left(\frac{x}{\sqrt{3}}\right) + C$

Remember:

$\int \frac{1}{x^2+a^2}\,dx$

$= \frac{1}{a}\arctan\left(\frac{x}{a}\right) + C$

Ex:

- $\int \frac{x^3+10x^2+3x+36}{(x-1)(x^2+4)^2}\,dx$

$\rightarrow \frac{x^3+10x^2+3x+36}{(x-1)(x^2+4)^2} = \frac{A}{x-1} + \frac{Bx+C}{x^2+4} + \frac{Dx+E}{(x^2+4)^2}$

$= A(x^2+4)^2 + (Bx+C)(x-1)(x^2+4) + (Dx+E)(x-1)$

$= A(x^4+16+8x^2) + (Bx+C)(x^3+4x-x^2-4) + (Dx+E)(x-1)$

$= x^4(A+B) + x^3(-B+C) + x^2(8A+4B-C+D) + x(-4B+4C-D+E)$

$+ (16A-4C-E)$

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 8 & 4 & -1 & 1 & 0 \\ 0 & -4 & 4 & -1 & 1 \\ 16 & 0 & -4 & 0 & -1 \end{bmatrix} \begin{bmatrix} A \\ B \\ C \\ D \\ E \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 10 \\ 3 \\ 36 \end{bmatrix} \Rightarrow \begin{matrix} A=2 \\ B=-2 \\ C=-1 \\ D=1 \\ E=0 \end{matrix}$$

$= \int \frac{2}{x-1} - \frac{2x+1}{x^2+4} + \frac{x}{(x^2+4)^2}\,dx$

$= 2\ln|x-1| - \int \frac{2x+1}{x^2+4}\,dx + \int \frac{x}{(x^2+4)^2}\,dx$

$-\int \frac{2x}{x^2+4} + \frac{1}{x^2+4}\,dx + \frac{1}{2}\int \frac{1}{u^2}\,dx$

$= 2\ln|x-1| - \ln|x^2+4| - \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) - \frac{1}{2}\frac{1}{x^2+4} + C$

Ex:

- $\int \frac{x^4-5x^3+6x^2-18}{x^3-3x^2}\,dx$

Long division!

$$\begin{array}{r} x^4-5x^3+6x^2+0x-18 \\ -x^4-3x^3 \\ \hline 0\ -2x^3+6x^2+0x-18 \\ -\ -2x^3+6x^2 \\ \hline 0\quad 0\quad 0x-18 \end{array} \quad \Big| \begin{array}{l} x^3-3x^2 \\ \hline x-2 \end{array}$$

$= \int (x-2) + \frac{-18}{x^2(x-3)}\,dx$

or $\frac{Ax+B}{x^2} + \frac{C}{x-3}$

$$\frac{18}{x^2(x-3)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-3}$$

$$= Ax(x-3) + B(x-3) + Cx^2$$

$$= A(x^2-3x) + B(x-3) + Cx^2$$

$$18 = x^2(A+C) + x(-3A+B) + (-3B)$$

$-3B = 18$ → $\boxed{B=-6}$ $\quad A+C=0$ → $A=-C$ → $\boxed{C=2}$ $\quad -3A+(-6)=0$ → $-3A=6$ → $\boxed{A=-2}$

$$= \int (x-2)\,dx - \int \frac{-2}{x} + \frac{-6}{x^2} + \frac{2}{x-3}\,dx$$

$$= \frac{1}{2}x^2 - 2x + 2\ln|x| - \frac{6}{x} - 2\ln|x-3| + C$$

! Estrategy:

1. Simplify: not necessarily small, but simpler to solve.
   Ex: $\int \sin^2(x)\,dx = \frac{1}{2}\int 1+\cos(2x)\,dx$
2. Don't forget Calc I! (sometimes u-sub is the easiest way out!)
3. Calc. II tecniques:
   a. (poly)$\begin{pmatrix}\sin x\\ \cos x\\ e^x\end{pmatrix}$ → integ by parts
   b. Trig. functions! → $\sin^n x\cos^m x$; $\sec^n x\tan^m x$; $\cot^n x \cdot \csc^m x$
   c. Trig. Substitutions → $\sqrt{b^2x^2 \pm a^2}$ $\quad \sqrt{a^2 \pm b^2x^2}$
   e. Partial Fractions: → $\frac{poly}{poly}$
4. Calc I: doing u-sub to then use a calc2 technique.
   Ex: $\int e^x\sqrt{1+e^{2x}}\,dx$; $\underset{du=e^x dx}{u=e^x} \longrightarrow \int \sqrt{1+u^2}\,du$ now we can do trig-sub.