

Partial Fractions

• To do it, we need to make sure of the following:

if $\frac{P(x)}{Q(x)}$, make sure $\deg(P(x)) < \deg(Q(x))$

Ex:

$$\int \frac{3x+11}{x^2-x-6} dx$$

→ DECOMP.:

$$\frac{3x+11}{(x-3)(x+2)} = \frac{A}{x-3} + \frac{B}{x+2}$$

$$3x+11 = A(x+2) + B(x-3)$$

$$= x(A+B) + (2A-3B)$$

$$A+B=3$$

$$A=3-B$$

$$A=3-(-1)$$

$$\boxed{A=4}$$

$$2A-3B=11$$

$$2(3-B)-3B=11$$

$$6-2B-3B=11$$

$$-5B=5 \rightarrow \boxed{B=-1}$$

Then:

$$= \int \frac{4}{x-3} - \frac{1}{x+2} dx = 4 \ln|x-3| - \ln|x+2| + C$$

Ex:

$$\int \frac{x^2+4}{3x^3+4x^2-4x} dx$$

DECOMPOSITION

$$\frac{x^2+4}{3x^3+4x^2-4x} = \frac{x^2+4}{x(x+2)(x-2)} = \frac{A}{x} + \frac{B}{x+2} + \frac{C}{x-2}$$

$$x^2+4 = A(x^2-4) + B(x^2-2x) + C(x^2+2x)$$

$$x^2+4 = x^2(A+B+C) + x(-2B+2C) + (-4A) \quad \forall x \in \mathbb{R}!$$

$$x=0: 4 = -4A \rightarrow \boxed{A=-1}$$

$$x=-2: 8 = B(-2)(-8) \rightarrow \boxed{B=1/2}$$

$$x=2/3: \frac{40}{9} = C\left(\frac{2}{3}\right)\left(\frac{8}{3}\right) \rightarrow \boxed{C=5/2}$$

$$= \int \frac{-1}{x} + \frac{1}{2} \left(\frac{1}{x+2} \right) + \frac{5}{2} \left(\frac{1}{x-2} \right) dx = -\ln|x| + \frac{1}{2} \ln|x+2| + \frac{5}{2} \ln|x-2| + C$$

$u=3x-2$
u-sub.

Ex

$$\int \frac{x^2-9x+5}{(x-4)^2(x^2+3)} dx$$

$$\hookrightarrow = \frac{A}{x-4} + \frac{B}{(x-4)^2} + \frac{Cx+D}{x^2+3}$$

$$A(x^2+3)(x-4) + B(x^2+3) + (Cx+D)(x-4)^2 = x^2-9x+5$$

$$A(x^3-4x^2+3x-12) + B(x^2+3) + (Cx+D)(x^2-8x+16) = x^2-9x+5$$

$$x^3(A+C) + x^2(-4A+B-8C) + x(3A+16C) + (-12A+3B+16D) = x^2-9x+5$$

$$A+C=0 \rightarrow A=-C$$

$$3(-C)+16C-8D=29$$

$$13C-8D=29 \rightarrow D=\frac{13C+29}{8}$$

$$\boxed{A=1 \quad B=-5}$$

$$\boxed{C=-1 \quad D=2}$$

