01-30-2019

Ex:

- $$\int_0^{1/6} \frac{x^5}{(36x^2+1)^{3/2}}\,dx = \ldots = \frac{1}{6^6}\int_0^{\pi/4} \frac{\tan^5\theta}{\sec\theta}\,d\theta$$

$$= \frac{1}{6^6}\int_0^{\pi/4} \frac{\sin^5\theta}{\cos^5\theta}\cdot\cos\theta\,d\theta = \frac{1}{6^6}\int_0^{\pi/4} \frac{\sin^5\theta}{\cos^4\theta}\,d\theta$$

$$= \frac{1}{6^6}\int_0^{\pi/4} \frac{(1-\cos^2\theta)^2}{\cos^4\theta}\sin\theta\,d\theta \qquad \begin{array}{l} u=\cos\theta \\ du=-\sin\theta\,d\theta \end{array}$$

$$= \frac{-1}{6^6}\int_0^{\pi/4} \frac{(1-u^2)^2}{u^4}\,du \qquad \begin{array}{l} u=\cos(\frac{\pi}{4})=\frac{\sqrt{2}}{2} \\ u=\cos(0)=1 \end{array}$$

$$= \frac{-1}{6^6}\int_0^{\pi/4} \frac{1+u^4-2u^2}{u^4}\,du = \frac{-1}{6}\int_0^{\pi/4} \frac{1}{u^4}+1-\frac{2}{u^2}\,du$$

$$= \frac{-1}{6^6}\left[-\frac{1}{3}u^{-3}+2u^{-1}+u\right]_0^{\pi/4} \rightarrow \theta \text{ limits}$$

$$= -\frac{1}{6^6}\left[-\frac{1}{3}(\cos\theta)^{-3}+2(\cos\theta)^{-1}+\cos\theta\right]_0^{\pi/4}$$

$$= \frac{1}{17496} - \frac{11\sqrt{2}}{279936}$$

Ex:

- $$\int \frac{x}{\sqrt{2x^2-4x-7}}\,dx$$ COMPLETING THE SQUARE!

$$2x^2-4x-7 = 2\left(x^2-2x-\frac{7}{2}\right) = 2\left(x^2-2x+1-1-\frac{7}{2}\right)$$

$a=x \quad -2x=2ab=2xb$

$b=-1$

$$= 2\left((x-1)^2-\frac{9}{2}\right)$$

$$= 2(x-1)^2-9$$

$$= \int \frac{x}{\sqrt{2(x-1)^2-9}}\,dx \qquad x-1=\frac{3}{\sqrt{2}}\sec\theta \qquad \sqrt{2\left(\frac{3}{\sqrt{2}}\sec\theta\right)^2-9} = \sqrt{9(\sec^2\theta-1)} = 3|\tan\theta| \text{ Assum. posit.}$$

$$1\cdot dx = \frac{3}{\sqrt{2}}\sec\theta\tan\theta\,d\theta \qquad = 3\tan\theta$$

$$\downarrow$$

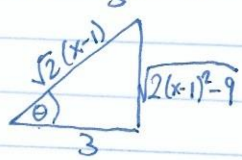
$$= \int \frac{\frac{3}{\sqrt{2}}\sec\theta+1}{\cancel{3}\cancel{\tan\theta}}\cdot\frac{\cancel{3}}{\sqrt{2}}\sec\theta\cancel{\tan\theta}\,d\theta$$

$$= \int \frac{1}{\sqrt{2}}\left(\frac{3}{\sqrt{2}}\sec\theta+1\right)\sec\theta\,d\theta = \int \frac{3}{2}\sec^2\theta+\frac{1}{\sqrt{2}}\sec\theta\,d\theta$$

Replace the $\theta \rightarrow x$! $$= \left[\frac{3}{2}\tan\theta+\frac{1}{\sqrt{2}}\ln|\sec\theta+\tan\theta|+C\right]$$

$$\downarrow\downarrow\downarrow$$

$$\sec\theta = \frac{\sqrt{2}}{3}(x-1)$$



$$\tan\theta = \frac{\sqrt{2(x-1)^2-9}}{3}$$

$$= \left[\frac{1}{2}\sqrt{2(x-1)^2-9}+\frac{1}{\sqrt{2}}\ln\left|\frac{\sqrt{2}(x-1)+\sqrt{2(x-1)^2-9}}{3}\right|+C\right]$$

Using Partial Fraction Decomposition

Ex:

- $\int \frac{2x-1}{x^2-x-6}\,dx$ $\quad u = x^2-x-6$, $\quad du = 2x-1\,dx$

$$= \int \frac{1}{u}\,du = \ln|x^2-x-6| + C$$

Ex:

- $\int \frac{3x+11}{x^2-x-6}\,dx =$ By Magic $= \int \frac{4}{x-3} - \frac{1}{x+2}\,dx = 4\ln|x-3| - \ln|x+2| + C.$

TABLE OF FACTORS:

$ax+b$	$\frac{A}{ax+b}$
$(ax+b)^n$	$\frac{A_1}{ax+b} + \frac{A_2}{(ax+b)^2} + \dots + \frac{A_n}{(ax+b)^n}$
ax^2+bx+c	$\frac{Ax+B}{ax^2+bx+c}$
$(ax^2+bx+c)^n$	$\frac{A_1x+B_1}{ax^2+bx+c} + \frac{A_2x+B_2}{(ax^2+bx+c)^2} + \dots + \frac{A_nx+B_n}{(ax^2+bx+c)^n}$