01-29-2019

HELP SESSION

■ Similar to #3:

○ $\int 2\tan^{-1}(4x)\,dx$ $\quad u = 2\tan^{-1}(4x)$ $\quad dv = dx$

$du = \frac{8}{1+(4x)^2}\,dx$ $\quad v = x$

$= 2x\tan^{-1}(4x) - 8\int \frac{x}{1+(4x)^2}\,dx$ $\quad u = 1+16x^2$ $\quad du = 32x\,dx$ $\quad dx = \frac{du}{32x}$

$= 2x\tan^{-1}(4x) - 8\int \frac{x}{u}\frac{du}{32x}$

$= 2x\tan^{-1}(4x) - \frac{1}{4}\int \frac{1}{u}\,du = 2x\tan^{-1}(4x) - \frac{1}{4}\ln|1+16x^2| + C$

■ Similar to #7:

○ $\int \cos^6(z)\,dz = \int (\cos^2(z))^3\,dz = \int \left[\frac{1}{2}(1+\cos(2z))\right]^3 dz$

$= \frac{1}{8}\int (1+\cos(2z))^3\,dz$

$= \frac{1}{8}\int 1 + 3\cos(2z) + 3\cos^2(2z) + \cos^3(2z)\,dz$

$= \frac{1}{8}\left[\int 1+3\cos(2z)\,dz + 3\int \cos^2(2z)\,dz + \int \cos^3(2z)\,dz\right]$

$\frac{3}{2}\int 1+\cos(4z)\,dz$

$\int \cos^2(2z)\cos(2z)\,dz$

$u = \sin(2z)$

$du = \cos(2z)\cdot 2\,dx$

$dx = \frac{du}{2\cos(2z)}$

$\int (1-\sin^2(2z))\cos(2z)\,dz$

$\int (1-u^2)\cancel{\cos(2z)}\frac{du}{\cancel{2\cos(2z)}}$

$\frac{1}{2}\int (1-u^2)\,du$

And you can continue...

■ Similar to #5:

○ $\int t^5 e^{2+t^3}\,dt = \int t^3 \cdot t^2 e^{2+t^3}\,dt$ $\quad u = t^3$ $\quad du = 3t^2\,dt$ $\quad dv = t^2 e^{2+t^3}\,dt$ $\quad v = \int t^2 e^{2+t^3}\,dt$

$= \int t^2 e^w \frac{dw}{3t^2}$ $\quad w = 2+t^3$ $\quad dw = 3t^2\,dt$ $\quad dt = \frac{dw}{3t^2}$

$= \frac{1}{3}\int e^w\,dw = \frac{1}{3}(e^{2+t^3})$

$= t^3 \cdot \frac{1}{3}\cdot e^{2+t^3} - \int t^2 e^{2+t^3}\,dt$

$= \frac{t^3}{3}e^{2+t^3} - \frac{1}{3}e^{2+t^3} + C$

$= \frac{e^{2+t^3}}{3}(t^3-1) + C$

Alternate way: $\int t^5 e^{2+t^3}\,dt$ $\quad w = 2+t^3 \rightarrow w-2 = t^3$ $\quad dw = 3t^2\,dt$

$= \frac{1}{3}\int t^3 e^w\,dw = \frac{1}{3}\int (w-2)e^w\,dw$ and then do integ. by parts.

■ Similar to #6:

- $\int e^x \sin^3(e^x)\cos^2(e^x)\,dx$ $\quad u = e^x$, $du = e^x dx \rightarrow \int \sin^3(u)\cos^2(u)\,du$

  and then continue...

Lecture:

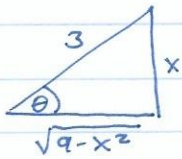
Ex:

- $\int \frac{1}{x^4\sqrt{9-x^2}}\,dx$ $\quad x = 3\sin\theta$

$= -\frac{1}{81}\left(\frac{1}{3}\cot^3\theta + \cot\theta\right) + C$ (now go back to x's)

$x = 3\sin\theta$

$\frac{x}{3} = \sin\theta$



Then: $\cot\theta = \frac{\sqrt{9-x^2}}{x}$

$= \frac{-1}{81}\left(\frac{1}{3}\left(\frac{\sqrt{9-x^2}}{x}\right)^3 + \frac{\sqrt{9-x^2}}{x}\right) + C$

Ex:

- $\int_0^{1/6} \frac{x^5}{(36x^2+1)^{3/2}}\,dx$ $\quad x = \frac{1}{6}\tan\theta$, $dx = \frac{1}{6}\sec^2\theta\,d\theta$

Limits:

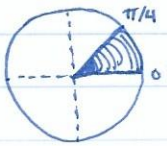
$x = 0 \rightarrow 0 = \frac{1}{6}\tan\theta$, $\tan\theta = 0$, $\theta = \tan^{-1}(0) = 0$

$x = 1/6 = \frac{1}{6}\tan\theta$, $\tan\theta = 1$, $\theta = \frac{\pi}{4}$

$(36x^2+1)^{3/2} = (\tan^2\theta + 1)^{3/2} = (\sec^2\theta)^{3/2}$

$= (\sqrt{\sec^2\theta})^3 = |\sec\theta|^3 = \sec^3\theta$

↓ pos. in 1st quad ------>



$= \int_0^{\pi/4} \frac{1}{6^5}\cdot\frac{\tan^5\theta}{\sec^3\theta}\,\frac{1}{6}\sec^2\theta\,d\theta$

$= \frac{1}{6^6}\int_0^{\pi/4} \frac{\tan^5\theta}{\sec\theta}\,d\theta$ sec doesn't work, then go to sines and cosines.

$= \frac{1}{6^6}\int_0^{\pi/4} \frac{\sin^5\theta}{\cos^5\theta}\cdot\cos\theta\,d\theta = \frac{1}{6^6}\int_0^{\pi/4} \frac{\sin^5\theta}{\cos^4\theta}\,d\theta$