01-25-2019

Ex:

- $\int \sec^4(x)\tan^6(x)dx$ $\quad u=\tan x \quad du=\sec^2 x\,dx$ $\quad \tan^2(x)+1=\sec^2(x)$

$= \int \sec^2(x)\tan^6(x)\sec^2(x)dx = \int (\tan^2(x)+1)\tan^6(x)\sec^2(x)dx$

$= \int (u^2+1)u^6\,du = \int u^8+u^6\,du = \frac{1}{9}\tan^9(x)+\frac{1}{7}\tan^7(x)+C$

Ex: • $\int \tan(x)\,dx = \int \frac{\sin(x)}{\cos(x)}dx$ $\quad u=\cos(x) \quad du=-\sin(x)dx \quad dx=\frac{du}{-\sin x}$

$= \int \frac{\cancel{\sin(x)}}{\cos(x)}\frac{du}{\cancel{-\sin(x)}} = -\int \frac{1}{u}du$

$= -\ln|\cos(x)|+C$

$= \ln|\cos(x)^{-1}|+C$

$= \ln|\sec(x)|+C$

Ex:

- $\int \tan^3(x)\,dx = \int \tan(x)\tan^2(x) = \int \tan x(\sec^2(x)-1)dx$

$= \int \tan x\sec^2 x\,dx - \int \tan x\,dx$

$u=\tan x$
$du=\sec^2 x\,dx$

$= \int u\,du - (\ln|\sec x|+c)$

$= \frac{1}{2}\tan^2 x - \ln|\sec x|+C$

Ex:

- $\int \sec(x)\,dx = \int \sec x)\cdot\frac{\sec x+\tan x}{\sec x+\tan x}dx$

$= \int \frac{\sec^2 x+\sec x\tan x}{\tan x+\sec x}dx$ $\quad u=\tan x+\sec x \quad du=\sec^2 x+\sec x\tan x\,dx$

$= \int \frac{1}{u}du$

$= \ln|\tan x+\sec x|+C$

Ex:

- $\int \sec^3(x)dx = \int \sec(x)\sec^2(x)dx$ $\quad u=\sec x \quad du=\sec x\tan x \quad dv=\sec^2 x\,dx \quad v=\tan x$

$= \sec x\tan x - \int \sec x\tan^2 x\,dx$

$= \sec x\tan x - \int \sec x(\sec^2 x-1)dx$

$\int \sec^3 x\,dx = \sec x\tan x - \int \sec^3 x\,dx + \int \sec x\,dx$

$2\int \sec^3 x\,dx = \sec x\tan x + \ln|\tan x+\sec x| + C$

$\int \sec^3 dx = \frac{1}{2}\sec x\tan x + \frac{1}{2}\ln|\tan x+\sec x| + C'$

Trig-Substitutions: (hard section)

Let's start with a Calc 1 problem:

$\int x\sqrt{25x^2-4}\,dx$ we would do $u = 25x^2-4 \to du = 50x\,dx$

Same with $\int \frac{x}{\sqrt{25x^2-4}}\,dx$.

But what if the problem was: $\int \frac{\sqrt{25x^2-4}}{x}\,dx$

- Don't ask why. Let $x = \frac{2}{5}\sec\theta$

Then ... $\Rightarrow \sqrt{25(\frac{2}{5}\sec\theta)^2-4} = \sqrt{4\sec^2\theta - 4} = 2\sqrt{\sec^2\theta - 1}$

$= 2\sqrt{\tan^2\theta} = 2|\tan\theta|$

Remember:

$\sqrt{4} = 2$, not -2. But if $x^2 = 4$, then $x = \pm 2$.

$\sqrt{(-2)^2} = \sqrt{4} = 2$